JEE MAIN · PAPER 1 · PHYSICS

18 Atoms & Nuclei

Formula & concept sheet

How the atom is built and how it radiates: Rutherford's nuclear atom, Bohr's model of hydrogen-like ions and the spectral series, then the nucleus itself: its size, mass defect and binding energy, fission and fusion, and a short note on radioactive decay. Each idea is explained first, then the formulas, shortcuts and traps.

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READ ALONGSIDE NCERT XII · Atoms; Nuclei

HOW TO USE THIS SHEET

1. **Skim** — Read the Key ideas, then the formula cards.
2. **Memorise** — Learn the formulas and check every Trap box.
3. **Test** — Do the worked examples, then this chapter's PYQs.
4. **Revise** — Use the last-minute checklist before the exam.

Formula memorise · **Key idea** why it works · **Trap** marks lost here · **PYQ** how it is asked

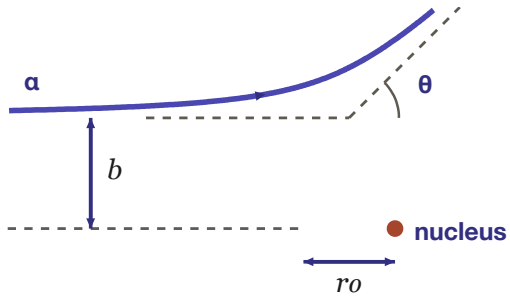
CHAPTER 18

Atoms & Nuclei

NCERT XII · Atoms; Nuclei

Covers: alpha-particle scattering and Rutherford's model · Bohr model and energy levels · hydrogen spectrum · composition and size of the nucleus · mass defect and binding energy · binding energy per nucleon curve · fission and fusion · radioactive decay (short note).

1 Rutherford's nuclear atom



impact parameter b, scattering angle θ and distance of closest approach r_0 (head-on, $b = 0$)

ALPHA-PARTICLE SCATTERING In Rutherford's experiment a beam of α particles hit a thin gold foil. Most passed almost straight through, a few were deflected through large angles, and about one in several thousand bounced almost straight back. So the positive charge and nearly all the mass sit in a tiny, dense **nucleus** (size about 10^{-15} m) while the atom is mostly empty space (size about 10^{-10} m).

WHAT DECIDES THE DEFLECTION

- The **impact parameter** b: how far the line of the incoming α misses the nucleus. Small b gives a large angle θ; $b = 0$ (head-on) gives $\theta = 180^\circ$.
- Repulsion between the α (charge $2e$) and the nucleus (charge Ze).

CLOSEST APPROACH (HEAD-ON)

$$r_0 = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{K}$$

all the initial kinetic energy K becomes Coulomb potential energy at the turning point; with $\frac{e^2}{4\pi\varepsilon_0} = 1.44$ MeV fm, $r_0 = \frac{2Z \times 1.44}{K(\text{MeV})}$ fm

IMPACT PARAMETER

$$b = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{K}\cot\frac{\theta}{2}$$

for an α particle; $b \to 0$ gives $\theta \to 180^\circ$

KEY IDEA

Rutherford's model has a nucleus with the electrons circling it, but it cannot explain why the atom is stable (a circling electron would radiate and spiral in) or why atoms emit only sharp lines. Bohr's postulates fix both.

QUICK CHECK

A 5 MeV α particle is fired head-on at gold ($Z = 79$). Closest approach?

Answer: $r_0 = \frac{2 \times 79 \times 1.44}{5} \approx 45$ fm.

2 The Bohr model

BOHR'S POSTULATES For a hydrogen-like atom (nucleus Ze, one electron) the electron moves in certain circular orbits without radiating, those with angular momentum $mvr = \dfrac{nh}{2\pi}$ ($n = 1, 2, 3, \ldots$). It radiates or absorbs a photon only when it jumps between orbits, with $hf = E_i - E_f$. It works for one-electron species only: H, He^{+}, Li^{2+}.

RADIUS AND ENERGY OF THE N-TH ORBIT

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

the Coulomb force supplies the centripetal force

$$mvr = \frac{nh}{2\pi}$$

the quantisation condition

$$r_n = \frac{n^2}{Z}a_0, \quad a_0 = 0.529\ \text{Å}$$

eliminate v

$$E_n = K + U = \frac{U}{2} = -K = -13.6\,\frac{Z^2}{n^2}\ \text{eV}$$

kinetic energy $K = -E$, potential energy $U = 2E$

RADIUS

$$r_n = 0.529\,\frac{n^2}{Z}\ \text{Å}$$

grows as n^2; the ground orbit of H is 0.529 Å

SPEED

$$v_n = 2.19 \times 10^6\,\frac{Z}{n}\ \text{m/s}$$

$= \dfrac{c}{137}\dfrac{Z}{n}$; falls as $\dfrac{1}{n}$

ENERGY

$$E_n = -13.6\,\frac{Z^2}{n^2}\ \text{eV}$$

negative (bound), rises towards 0 as n grows; $K = -E$, $U = 2E$

SCALINGS

$$r \propto \frac{n^2}{Z},\ v \propto \frac{Z}{n},\ E \propto \frac{Z^2}{n^2}$$

take ratios to remove all constants

PERIOD AND FREQUENCY

$$T \propto \frac{n^3}{Z^2},\quad f \propto \frac{Z^2}{n^3}$$

from $T = \dfrac{2\pi r}{v}$

IONISATION ENERGY

$$13.6\,\frac{Z^2}{n^2}\ \text{eV}$$

from level n to ∞; H ground 13.6 eV, He^{+} ground 54.4 eV, first excitation of H 10.2 eV

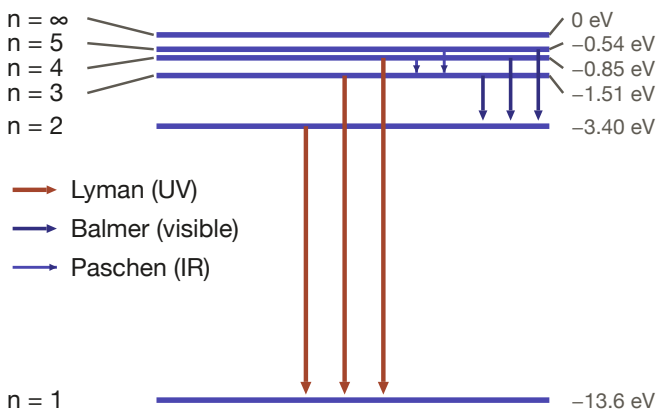
WHY IT WORKS

Quantisation is a standing wave: de Broglie's $\lambda = h/mv$ with $mvr = nh/2\pi$ gives $2\pi r = n\lambda$, a whole number of wavelengths round the orbit. See **Dual Nature**.

TRAP

Radius goes as n^2/Z, speed as Z/n (not Z/n^2), energy as Z^2/n^2. $n = 2$ is the **first** excited state. Energy levels are negative: "energy needed" questions take the magnitude.

3 The hydrogen spectrum



lines of each series are the transitions to $n_1 = 1,\ 2,\ 3$: $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n^2}\right)$

- A jump from a higher level n_2 to a lower one n_1 emits a photon of energy $\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ eV, with $\lambda(\text{nm}) = \frac{1240}{\Delta E(\text{eV})}$.
- All lines ending on the same lower level n_1 form a **series**: the first line (longest λ) has $n_2 = n_1 + 1$, and the series limit (shortest λ) has $n_2 = \infty$.
- Absorption is the reverse: the atom takes exactly the energy difference.

SPECTRAL LINE

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$R = 1.097 \times 10^7\ \text{m}^{-1}$, $n_2 > n_1$

NUMBER OF LINES

$$\frac{n(n-1)}{2}$$

from level n to the ground, counting every path; one atom emits fewer in a single cascade

PHOTONS IN A CASCADE

$$\sum E_{photon} = E_{start} - E_{end}$$

the photon energies add up to the total drop

SERIES OF HYDROGEN (Z = 1)

SERIES	n_1	REGION	FIRST LINE (NM)	SERIES LIMIT (NM)
Lyman	1	ultraviolet	121.5	91.2
Balmer	2	visible	656.3	364.6
Paschen	3	infrared	1875	820.4
Brackett	4	infrared	4051	1458
Pfund	5	infrared	7458	2279

Series limit $= \frac{n_1^2}{R}$. For a hydrogen-like ion of charge Z, every wavelength is the hydrogen value divided by Z^2 (He^+ Lyman limit 22.8 nm).

KEY IDEA

Ratio of two lines of the same series needs no R: $\frac{\lambda_{4\to2}}{\lambda_{3\to2}} = \frac{1/4 - 1/9}{1/4 - 1/16} = \frac{20}{27}$, so 656 nm becomes about 486 nm. He^+ $4 \to 2$ has the same wavelength as H $2 \to 1$: both give $\frac{3}{4}R$.

TRAP

Forgetting Z^2 in $1/\lambda$ for He^+ and Li^{2+} is the commonest slip. 13.6 eV is the ionisation energy of **hydrogen**; of He^+ it is 54.4 eV.

4 The nucleus: size, mass defect and binding energy

COMPOSITION A nucleus of Z protons and $N = A - Z$ neutrons has mass number A. **Isotopes** share Z, **isobars** share A, **isotones** share N. The strong nuclear force that holds it together is attractive, very short-ranged (a few fm), and the same between any two nucleons, charged or not.

RADIUS

$$R = R_0 A^{1/3}$$

$R_0 \approx 1.2$ fm; the volume is proportional to A

DENSITY

$$\rho \approx 10^{17} \text{ kg/m}^3$$

the same for every nucleus (about 2.3×10^{17} for $R_0 = 1.2$ fm), since volume and mass both go as A

ATOMIC MASS UNIT

$$1 \text{ u} = 931.5 \text{ MeV}/c^2$$

$= 1.66 \times 10^{-27}$ kg; $m_p = 1.007276$ u, $m_n = 1.008665$ u, $m_H = 1.007825$ u

MASS DEFECT

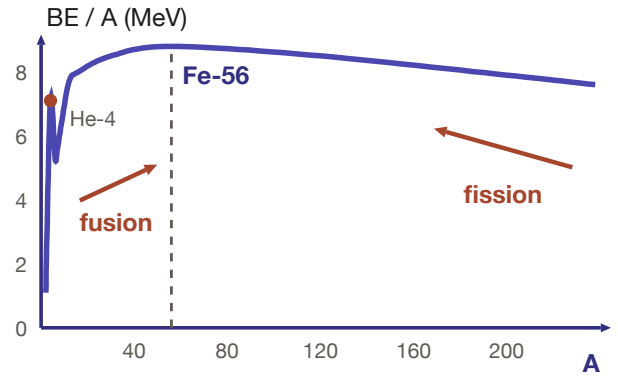
$$\Delta m = Z m_p + (A - Z) m_n - M_{nucleus}$$

with **atomic** masses use m_H in place of m_p: the electron masses then cancel

BINDING ENERGY

$$BE = \Delta m\, c^2$$

the energy released when the nucleus forms, or needed to pull it apart; BE/A per nucleon measures stability



the peak near iron explains why both fusion of light nuclei and fission of heavy nuclei release energy

BINDING ENERGY PER NUCLEON

- Rises steeply for light nuclei, peaks near $A \approx 56$ to 62 (iron and nickel, about 8.8 MeV per nucleon), then falls slowly to about 7.6 MeV for the heaviest nuclei.
- **Fusion** of light nuclei and **fission** of heavy ones both end on nuclei with higher BE/A, so both release energy.
- Higher BE/A means more stable, not larger total binding energy.

Q VALUE

$$Q = (\sum m_i - \sum m_f)c^2$$

$= \sum BE_f - \sum BE_i$; $Q > 0$ releases energy; balance A and Z first

FISSION OR FUSION FROM PER-NUCLEON VALUES

$$Q = \sum A_i B_i(\text{products}) - AB(\text{parent})$$

multiply each BE/A by its mass number; no masses needed

TWO-BODY SPLIT AT REST

$$K_1 = Q\,\frac{m_2}{m_1 + m_2}$$

momenta equal and opposite, $\frac{K_1}{K_2} = \frac{m_2}{m_1}$: the lighter piece takes more energy

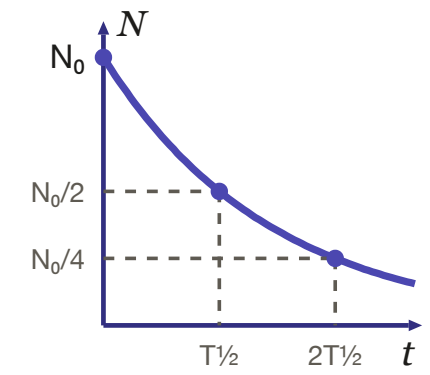
KEY IDEA

Typical energies: fission of ^{235}U gives about 200 MeV; D-T fusion, $^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$, about 17.6 MeV, but far more **per nucleon** than fission.

TRAP

Sum the mass defect in u to six decimals and convert once with 931.5 MeV. With atomic masses use m_H, not m_p, or the answer is off by an electron mass per proton. Alpha decay of a parent A at rest: $K_\alpha = Q\,\dfrac{A-4}{A}$.

5 Radioactive decay (short note)



$N = N_0 e^{-\lambda t}$, every $T_{1/2}$ halves N

- Unstable nuclei decay at random; each nucleus has the same chance per second, λ (the decay constant), so the number N left falls exponentially.
- α **decay**: A falls by 4, Z by 2. β^-: Z rises by 1, A unchanged. β^+: Z falls by 1 . γ: neither changes. Balance A and Z to name the particle.

DECAY LAW

$N = N_0 e^{-\lambda t}$

after $n = \dfrac{t}{T_{1/2}}$ half-lives, $N = N_0 2^{-n}$

HALF-LIFE AND MEAN LIFE

$T_{1/2} = \dfrac{\ln 2}{\lambda} = 0.693\,\tau, \quad \tau = \dfrac{1}{\lambda}$

mean life is longer than half-life by $\dfrac{1}{0.693}$

ACTIVITY

$\mathcal{A} = \lambda N = \mathcal{A}_0 e^{-\lambda t}$

becquerel (decays per second); 1 Ci $= 3.7 \times 10^{10}$ Bq

SYLLABUS NOTE

Radioactivity is not in the topics listed for this chapter, but decay questions have appeared in recent papers, so the essentials are kept here.

6 Worked examples

LI^{2+} PHOTON

Wavelength of the photon emitted in the transition $n = 3 \rightarrow 1$ in Li^{2+} ($Z = 3$)?

$\Delta E = 13.6 \times 9\left(1 - \dfrac{1}{9}\right) = 108.8$ eV; $\lambda = \dfrac{1240}{108.8} \approx 11.4$ nm

RATIOS FOR IONS

Compare He$^+$ in $n = 2$ with H in $n = 1$: ratio of orbit radii and of electron speeds.

$r \propto \dfrac{n^2}{Z}$: $\dfrac{4/2}{1/1} = 2$; $v \propto \dfrac{Z}{n}$: $\dfrac{2/2}{1/1} = 1$, so equal speeds

COUNTING LINES

Lines a hydrogen atom can emit in falling from $n = 4$ to the ground state (all paths)?

$$\frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$$

BALMER LINE

The $3 \to 2$ Balmer line is 656 nm. Wavelength of the $4 \to 2$ line?

$$656 \times \frac{20}{27} \approx 486 \text{ nm}$$

BINDING ENERGY OF ^{4}HE

Atomic masses: ^{4}He 4.002603 u, ^{1}H 1.007825 u, neutron 1.008665 u. BE and BE/A?

$\Delta m = 2(1.007825) + 2(1.008665) - 4.002603 = 0.030377$ u; $BE = 0.030377 \times 931.5 \approx 28.3$ MeV, 7.07 MeV per nucleon

FISSION ENERGY

A nucleus with $A = 220$ and 5.6 MeV per nucleon splits into fragments of $A = 105$ and 115, each at 6.4 MeV per nucleon. Energy released?

$$Q = 220 \times 6.4 - 220 \times 5.6 = 176 \text{ MeV}$$

FUSION OF DEUTERONS

Two deuterons (1.1 MeV per nucleon) fuse into ^{4}He (7 MeV per nucleon). Energy released?

$$Q = 4 \times 7 - 2 \times 2 \times 1.1 = 23.6 \text{ MeV}$$

ALPHA DECAY ENERGY

^{238}U decays at rest with $Q = 4.27$ MeV. Kinetic energy of the α?

$$K_\alpha = Q\,\frac{A-4}{A} = 4.27 \times \frac{234}{238} \approx 4.20 \text{ MeV}$$

HALF-LIVES

Half-life 10 h: fraction left after 30 h. And the time for 75% of a sample with half-life 2 h to decay?

30 h is 3 half-lives: $2^{-3} = \frac{1}{8}$; 75% decayed leaves $\frac{1}{4} = 2^{-2}$: 4 h

MEAN LIFE

A source has mean life 10 h. Its half-life?

$$T_{1/2} = 0.693\,\tau \approx 6.93 \text{ h}$$

7 Last-minute checklist

BEFORE YOU ANSWER

Which ion (Z) and which levels (n)? Magnitude or sign of the energy? Atomic or nuclear masses? Is a half-life or a decay constant given?

- Rutherford: $r_0 = \dfrac{2Ze^2}{4\pi\varepsilon_0 K}$ $(= 2Z \times 1.44/K(\text{MeV})$ fm); $b \to 0$ means $\theta \to 180^\circ$.
- Bohr: $mvr = nh/2\pi$; $r \propto n^2/Z$, $v \propto Z/n$, $E = -13.6Z^2/n^2$ eV; $K = -E$, $U = 2E$.
- $1/\lambda = RZ^2(1/n_1^2 - 1/n_2^2)$; $\lambda = 1240/\Delta E$ nm; lines $n(n-1)/2$; first line $n_2 = n_1 + 1$, limit $n_2 = \infty$.
- $R = R_0 A^{1/3}$, density the same for all nuclei; Δm in u times 931.5 MeV; use m_H with atomic masses.
- BE/A peaks near iron; $Q = \sum BE_f - \sum BE_i$; fission and fusion both release energy.
- $N = N_0 e^{-\lambda t}$; $T_{1/2} = 0.693/\lambda$; α lowers A by 4 and Z by 2.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**