JEE MAIN · PAPER 1 · PHYSICS

17 Dual Nature of Matter & Radiation

Formula & concept sheet

Light behaves as a wave in interference and as a stream of photons in the photoelectric effect, and matter has a wavelength too. The sheet builds the photon picture, explains the photoelectric effect and its graphs from Einstein's equation, then gives the de Broglie wavelength for every case a numerical can use. Each idea is explained first, then the formulas, shortcuts and traps.

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READ ALONGSIDE NCERT XII · Dual Nature of Radiation and Matter

HOW TO USE THIS SHEET

1. **Skim** — Read the Key ideas, then the formula cards.
2. **Memorise** — Learn the formulas and check every Trap box.
3. **Test** — Do the worked examples, then this chapter's PYQs.
4. **Revise** — Use the last-minute checklist before the exam.

CHAPTER 17

Dual Nature of Matter & Radiation

NCERT XII · Dual Nature of Radiation and Matter

Covers: photons · photoelectric effect and the observations of Hertz and Lenard · work function, threshold, stopping potential · Einstein's equation · photocurrent and stopping-potential graphs · matter waves and the de Broglie wavelength.

1 Light as photons

PHOTON Light of frequency f is absorbed and emitted in packets of energy $E = hf$ (Planck's constant $h = 6.626 \times 10^{-34}$ J s). A photon has no rest mass, moves at c, and carries momentum $p = \dfrac{E}{c} = \dfrac{h}{\lambda}$. The intensity of a beam is the number of photons crossing unit area per second times the energy of one photon, so doubling the intensity doubles the photons, not the energy of each.

PHOTON ENERGY

$$E = hf = \frac{hc}{\lambda}$$

$E(\text{eV}) = \dfrac{1240}{\lambda(\text{nm})}$; papers use 1237, 1240 or 1242: use the value printed in the question

PHOTON MOMENTUM

$$p = \frac{h}{\lambda} = \frac{E}{c}$$

no mass formula: for a photon $E = pc$

PHOTONS PER SECOND

$$n = \frac{P}{E} = \frac{P\lambda}{hc}$$

source of power P; with intensity I on area A: $\dfrac{IA}{hf}$ per second

KEY IDEA

At equal power, a source of longer wavelength emits **more** photons, since each carries less energy: the photon count is proportional to λ (300 nm and 500 nm give $3 : 5$).

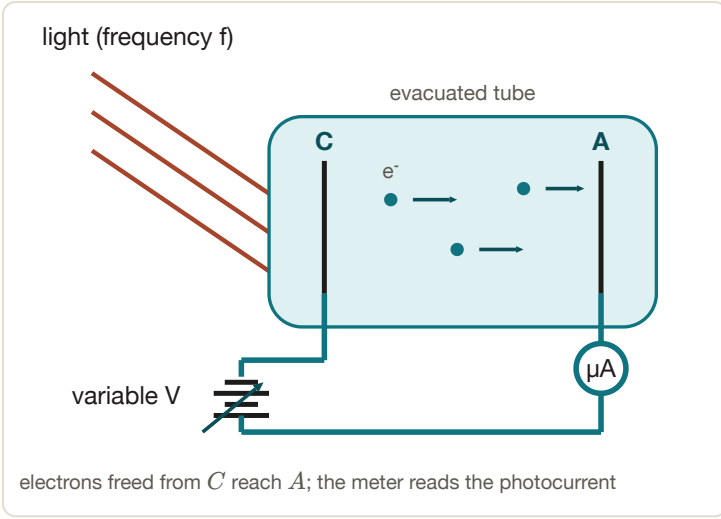
QUICK CHECK

A 1 mW laser emits light of wavelength 632.8 nm. Photons per second?

Answer: $n = \dfrac{P\lambda}{hc} = \dfrac{10^{-3} \times 632.8 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^{8}} \approx 3.2 \times 10^{15}$ per second.

2 The photoelectric effect

PHOTOELECTRIC EFFECT When light of high enough frequency falls on a metal surface, electrons are ejected. The **work function** ϕ is the least energy needed to free an electron from the metal (it depends on the metal). The **threshold frequency** $f_0 = \dfrac{\phi}{h}$ is the lowest frequency that works.



electrons freed from C reach A; the meter reads the photocurrent

HERTZ AND LENARD: WHAT IS OBSERVED

- There is a **threshold frequency** f_0 for each metal: below it no electrons come out, however intense the light or long the exposure.
- Above f_0 emission is **instantaneous** (no time lag), even for very faint light.
- The photocurrent is proportional to the **intensity**; the maximum kinetic energy of the electrons does **not** depend on intensity, only on the frequency.
- A reverse (retarding) voltage V_0, the **stopping potential**, just stops the fastest electrons: $K_{max} = eV_0$.

Why the wave picture fails. A wave would give each electron energy that grows with intensity, let a dim beam work slowly after a delay, and let any frequency work given time. None of this is seen. Einstein's photon picture: **one photon gives all its energy to one electron** in one event; if $hf < \phi$ the electron cannot escape, if $hf > \phi$ the surplus is its kinetic energy.

WHY IT WORKS

Energy is conserved for each photon-electron pair: $hf = \phi + K$. Electrons deeper in the metal lose some energy on the way out, so K_{max} is the energy of the **fastest** electrons; most leave slower.

TRAP

Below the threshold there is no current at all, not a small one. Emission needs **frequency** above f_0; making a low-frequency beam brighter does nothing.

3 Einstein's photoelectric equation

EINSTEIN'S EQUATION

$$K_{max} = hf - \phi = eV_0$$

ϕ and K_{max} in the same unit (eV or J)

THRESHOLD

$$f_0 = \frac{\phi}{h}, \quad \lambda_0 = \frac{hc}{\phi}$$

at $f = f_0$, $K_{max} = 0$; $\lambda_0(\text{nm}) = \dfrac{1240}{\phi(\text{eV})}$

WAVELENGTH FORM

$$K_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

in eV: $1240\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$ with λ in nm

STOPPING POTENTIAL

$$V_0 = \frac{h}{e}f - \frac{\phi}{e}$$

straight line in f: slope $\frac{h}{e}$ for every metal, intercept on the f axis f_0

FASTEST ELECTRON

$$v_{max} = \sqrt{\frac{2K_{max}}{m_e}}$$

non-relativistic; K_{max} in joules

VOLTS DIRECTLY

$$V_0(\text{V}) = \frac{1240}{\lambda(\text{nm})} - \phi(\text{eV})$$

the stopping potential in volts has the same number as K_{max} in eV

PHOTOELECTRIC NUMERICALS

1. Photon energy in eV: $E = 1240/\lambda(\text{nm})$ (or hf with $h = 4.136 \times 10^{-15}$ eV s).
2. $K_{max} = E - \phi$ in eV; this is also V_0 in volts. If $E < \phi$ the answer is: no emission.
3. Two readings of V_0 (two wavelengths or frequencies): subtract the two Einstein equations; ϕ cancels and leaves $\frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = V_{01} - V_{02}$.
4. Only a fraction of photons free an electron: multiply the photon rate by that efficiency to get the number of electrons.

TRAP

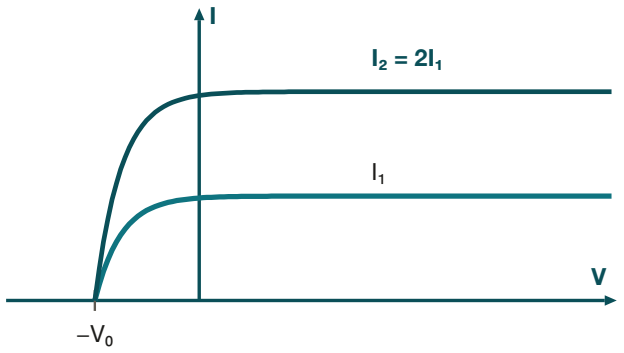
Doubling the frequency does **not** double K_{max}: ϕ is subtracted first. Convert ϕ and K to the same unit before subtracting, and keep 1 eV $= 1.6 \times 10^{-19}$ J.

QUICK CHECK

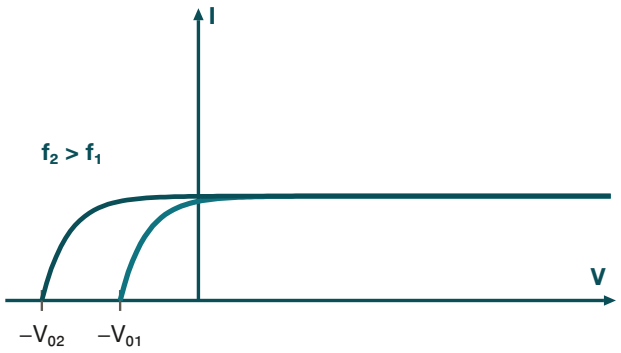
Threshold wavelength 380 nm; light of 260 nm falls on the metal. Maximum kinetic energy?

Answer: $1240\left(\frac{1}{260} - \frac{1}{380}\right) \approx 1.5$ eV.

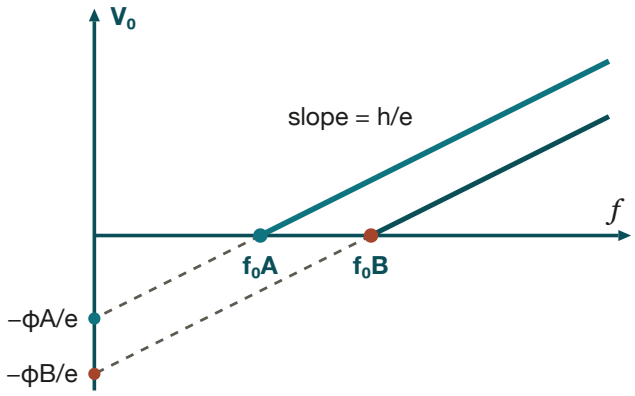
4 Photocurrent and stopping-potential graphs



same frequency, double intensity: saturation current doubles, V_0 same



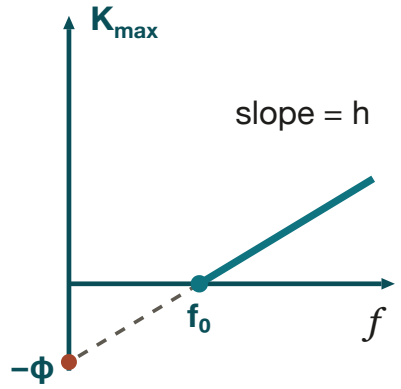
same intensity, higher frequency: larger V_0, same saturation current



parallel lines for every metal; the f-axis intercept is the threshold f_0

READING THE GRAPHS

- I **against** V: the current is zero at $V = -V_0$, rises with the anode voltage and flattens at the **saturation current**, when every emitted electron is collected.
- Saturation current depends on **intensity** (the number of photons per second); the stopping potential depends on **frequency** and the metal.
- V_0 **against** f: a straight line of slope h/e (the same for every metal), with an intercept $-\phi/e$ on the voltage axis and f_0 on the frequency axis. Two metals give **parallel** lines; the one with the larger ϕ is shifted to the right.



$K_{\max} = hf - \phi$: slope h, intercept $-\phi$

WHAT CHANGES WHAT

IF YOU...	SATURATION CURRENT	K_{max} AND V_0
double the intensity (same f)	doubles	unchanged
raise the frequency (same photon rate)	unchanged	increase
use a metal with a larger ϕ (same f)	unchanged	decrease; may stop emission

SLOPE TRICK

$h = e\times$ slope of the V_0-f line

two points on the line give h without knowing the metal; K_{max} against f has slope h directly

TRAP

The x-intercept of the V_0 graph is the **threshold frequency**, where the stopping potential is zero. The stopping potential itself is read on the voltage axis. Retarding voltages are drawn negative; the stopping potential is the magnitude.

5 Matter waves: the de Broglie wavelength

DE BROGLIE HYPOTHESIS If light, a wave, behaves as particles with momentum $p = h/\lambda$, then particles should behave as waves: a particle of momentum p has the wavelength $\lambda = \dfrac{h}{p}$. It is only noticeable for the tiny momenta of atomic particles; a 1 kg object at 1 m/s has $\lambda \approx 6.6 \times 10^{-34}$ m.

THE WAVELENGTH IN EACH SITUATION

SITUATION	WAVELENGTH	NOTES
Momentum p or speed v	$\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$	any particle
Kinetic energy K	$\lambda = \dfrac{h}{\sqrt{2mK}}$	$K = \dfrac{h^2}{2m\lambda^2}$
Charge q accelerated through V from rest	$\lambda = \dfrac{h}{\sqrt{2mqV}}$	electron: $\dfrac{1.227}{\sqrt{V}}$ nm; proton: $\dfrac{0.0286}{\sqrt{V}}$ nm (V in volts)
Thermal particle at temperature T	$\lambda = \dfrac{h}{\sqrt{3mk_BT}}$	the value given in the question wins; some books use $\sqrt{2\pi mk_BT}$
Photon	$\lambda = \dfrac{h}{p} = \dfrac{hc}{E}$	never $h/\sqrt{2mK}$; for a photon $E = pc$

SAME MOMENTUM

λ equal for any mass or charge

kinetic energies then go as $\frac{1}{m}$: $\frac{K_p}{K_e} = \frac{m_e}{m_p}$

SAME KINETIC ENERGY

$$\lambda \propto \frac{1}{\sqrt{m}}$$

electron to proton $\sqrt{1836} \approx 43$; $\lambda_p : \lambda_d : \lambda_\alpha = 1 : \frac{1}{\sqrt{2}} : \frac{1}{2}$

SAME ACCELERATING VOLTAGE

$$\lambda \propto \frac{1}{\sqrt{mq}}$$

$\lambda_p : \lambda_d : \lambda_\alpha = 1 : \frac{1}{\sqrt{2}} : \frac{1}{\sqrt{8}}$

ELECTRON AND PHOTON OF THE SAME λ

$$\frac{K_e}{E_{ph}} = \frac{v}{2c}$$

non-relativistic: $\frac{p^2/2m}{pc}$; at $v = 0.25c$ the ratio is $\frac{1}{8}$

SAME SPEED

$$\lambda \propto \frac{1}{m}$$

for the same v, a heavier particle has a shorter wavelength

HEAD-ON, THEN STUCK TOGETHER

$$\lambda = \frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$$

momenta add as vectors, so $\frac{1}{\lambda} = \left|\frac{1}{\lambda_x} - \frac{1}{\lambda_y}\right|$

TRAP

The formula $1.227/\sqrt{V}$ nm is for **electrons** and slow (non-relativistic) speeds only. For an α particle at the same V the wavelength is $\frac{1}{\sqrt{8}}$ of the proton's, because it has $4\times$ the mass and $2\times$ the charge.

CONNECTS TO

Matter waves explain Bohr's quantisation: a stable electron orbit holds a whole number of wavelengths, $2\pi r = n\lambda$, which gives $L = n\hbar$. See **Atoms and Nuclei**.

SYLLABUS NOTE

X-ray production, Compton scattering and the Davisson-Germer experiment are discussed in textbooks but are not in the topics listed for this chapter, so they are left out here.

6 Worked examples

MAXIMUM KINETIC ENERGY

The threshold wavelength is 380 nm and the light has 260 nm. K_{max}?

$$K_{max} = 1240\left(\frac{1}{260} - \frac{1}{380}\right) \approx 1.5 \text{ eV}$$

THRESHOLD WAVELENGTH

A metal has $\phi = 2.14$ eV and a stopping potential of 2 V is measured. The wavelength used?

$hf = 2.14 + 2 = 4.14$ eV, so $\lambda = 1240/4.14 \approx 300$ nm

WORK FUNCTION FROM TWO READINGS

V_0 is 8 V for wavelength λ and 2 V for 3λ. Find ϕ and the threshold wavelength.

$\frac{hc}{\lambda} - \phi = 8$, $\frac{hc}{3\lambda} - \phi = 2$; subtract: $\frac{2}{3}\frac{hc}{\lambda} = 6$, $\frac{hc}{\lambda} = 9$ eV, $\phi = 1$ eV, $\lambda_0 = 1240$ nm

THRESHOLD FREQUENCY

A metal's work function is 6.63 eV. Threshold frequency?

$$f_0 = \frac{6.63 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.6 \times 10^{15} \text{ Hz}$$

WORK FUNCTION FROM V_0

Light of 1.2×10^{15} Hz gives a stopping potential of 2.5 V. Find ϕ.

$hf = 4.136 \times 10^{-15} \times 1.2 \times 10^{15} = 4.96$ eV; $\phi = 4.96 - 2.5 \approx 2.46$ eV

PHOTONS FROM TWO SOURCES

Two 200 W sources emit 300 nm and 500 nm light. Ratio of photons per second?

$n = \dfrac{P\lambda}{hc} \propto \lambda$: $n_{300} : n_{500} = 3 : 5$

ELECTRON THROUGH 100 V

de Broglie wavelength of an electron accelerated through 100 V?

$\lambda = \dfrac{1.227}{\sqrt{100}} = 0.123$ nm

PROTON AND α

A proton has λ and an α particle 2λ. Ratio of their speeds?

$v \propto \dfrac{1}{m\lambda}$: $\dfrac{v_p}{v_\alpha} = \dfrac{4m_p \times 2\lambda}{m_p \times \lambda} = 8$

ELECTRON AND PHOTON

An electron at $0.25c$ has the same wavelength as a photon. Ratio of their energies?

$\dfrac{K_e}{E_{ph}} = \dfrac{v}{2c} = \dfrac{1}{8}$

EQUAL KINETIC ENERGY

An electron and a proton have the same kinetic energy. Ratio of the wavelengths?

$\dfrac{\lambda_e}{\lambda_p} = \sqrt{\dfrac{m_p}{m_e}} \approx 43$

7 Last-minute checklist

BEFORE YOU ANSWER

Is hf above the work function at all? Which hc value does the question use? Is a graph asking about intensity (current) or frequency (stopping potential)? Is the particle an electron, a proton or a photon?

- $E = hf = hc/\lambda = 1240/\lambda(\text{nm})$ eV; $p = h/\lambda = E/c$; photons per second $P\lambda/hc$.
- $K_{max} = hf - \phi = eV_0$; threshold $f_0 = \phi/h$; below it, no emission at any intensity.
- Intensity sets the photocurrent; frequency sets K_{max} and V_0; emission is instantaneous.
- V_0-f line: slope h/e (same for all metals), intercept on the f axis f_0; two metals give parallel lines.
- $\lambda = h/p = h/\sqrt{2mK} = h/\sqrt{2mqV}$; electron $1.227/\sqrt{V}$ nm; same momentum means same λ.
- Photon: $E = pc$, never the mass formula; electron and photon of the same λ: $K_e/E = v/2c$.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**