JEE MAIN · PAPER 1 · PHYSICS

15

Electromagnetic Waves

Formula & concept sheet

Maxwell's displacement current completes Ampere's law and predicts waves of electric and magnetic fields. The sheet explains displacement current, what an electromagnetic wave is and how its fields are related, how it carries energy and momentum, and the electromagnetic spectrum from radio to gamma rays. Each idea is explained first, then the formulas, shortcuts and traps.

INSIDE THIS SHEET

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READ ALONGSIDE NCERT XII · Electromagnetic Waves

HOW TO USE THIS SHEET

1. **Skim** — Read the Key ideas, then the formula cards.
2. **Memorise** — Learn the formulas and check every Trap box.
3. **Test** — Do the worked examples, then this chapter's PYQs.
4. **Revise** — Use the last-minute checklist before the exam.

Formula memorise · **Key idea** why it works · **Trap** marks lost here · **PYQ** how it is asked

Practise this chapter with real past-paper questions on Prep Vault. Written for revision, not as a first read.

CHAPTER 15

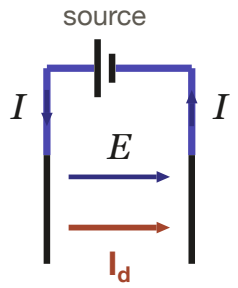
Electromagnetic Waves

NCERT XII · Electromagnetic Waves

Covers: displacement current · Ampere-Maxwell law · electromagnetic waves and their transverse nature · E = cB · energy, intensity, momentum and radiation pressure · the electromagnetic spectrum and its uses.

1 Displacement current and Maxwell's correction

DISPLACEMENT CURRENT While a capacitor charges, a current I flows in the wires but no charge crosses the gap between the plates. Maxwell noticed that the electric flux in the gap is changing, and defined a **displacement current** $I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$ there. It is not a flow of charge: it is a changing electric field that acts, for magnetism, like a current.



no conduction current
in the gap, only changing E
$I_d = \varepsilon_0\, d\Phi/dt$
($I_d = I$)

Inside the gap the changing E carries I_d, which continues the wire current I

- Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ gives different answers for two surfaces bounded by the same loop, one through the wire and one bulging through the gap. Adding I_d removes the contradiction.
- In the gap I_d **equals** the wire current I, so the total current is continuous round the circuit.
- I_d exists only while the field is changing: it is zero for a steady DC field.
- The other half of the story: a changing $\vec{E}$ produces a $\vec{B}$ (this correction), and a changing $\vec{B}$ produces an $\vec{E}$ (Faraday). Together they let the fields sustain each other as a wave.

DISPLACEMENT CURRENT

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

for plates of area A: $I_d = \varepsilon_0 A \frac{dE}{dt} = C\frac{dV}{dt}$

AMPERE-MAXWELL LAW

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

I_c = conduction current through the loop

AC ACROSS A CAPACITOR

$$I_{d,max} = V_0\, \omega C$$

$\omega = 2\pi f$; equals the peak current in the wires

TRAP

Displacement current is not charge crossing the gap, and it does not exist in a steady DC circuit. Between the plates the conduction current is zero; I_d takes over.

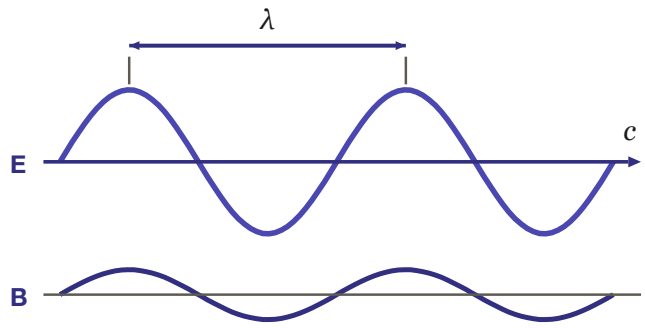
QUICK CHECK

A $12\ \mu$F capacitor has an AC voltage of amplitude 40 V, 4 kHz across it. Maximum displacement current between the plates?

Answer: $V_0 \omega C = 40 \times 2\pi \times 4000 \times 12 \times 10^{-6} \approx 12$ A.

2 Electromagnetic waves: nature, speed and field relation

ELECTROMAGNETIC WAVE A wave of oscillating electric and magnetic fields that sustain each other and travel through empty space. It is made by **accelerated charges** and needs no medium. $\vec{E}$, $\vec{B}$ and the direction of travel are mutually perpendicular, so the wave is **transverse**.



E × B points along the direction of travel

E and B rise and fall together, both perpendicular to the direction of travel

PLANE WAVE

- $\vec{E}$ and $\vec{B}$ are **in phase**: they reach their maximum and zero together, and the wave travels along $\vec{E} \times \vec{B}$.
- The wave carries no charge, so electric and magnetic fields do not deflect it.
- The direction of $\vec{E}$ is the direction of polarisation of the wave.
- On entering a medium the frequency (fixed by the source) does not change; the speed and the wavelength drop by the factor n.

SPEED IN VACUUM

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

$c = f\lambda = \frac{\omega}{k}$; Maxwell's theory gave this speed, which matched light

SPEED IN A MEDIUM

$$v = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{c}{n}$$

$n = \sqrt{\mu_r \varepsilon_r}$; for a non-magnetic medium $n = \sqrt{\varepsilon_r}$

FIELD RELATION

$$\frac{E_0}{B_0} = c, \quad E = cB$$

holds for peak and for instantaneous values; in a medium $E/B = v$

WAVE FORM

$$E = E_0 \sin(kz - \omega t), \quad B = B_0 \sin(kz - \omega t)$$

$k = \frac{2\pi}{\lambda}$; $kz - \omega t$ travels towards $+z$, $kz + \omega t$ towards $-z$

DIRECTION OF $\vec{B}$ FROM $\vec{E}$

1. Read the direction of travel from the sign in $kz \mp \omega t$ (minus: $+z$).
2. Pick $\hat{B}$ so that $\hat{E} \times \hat{B}$ points along the travel direction.
3. Use the cyclic triple, $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$.
4. Magnitude: $B_0 = E_0/c$.

TRAP

A wrong sign in $kz \pm \omega t$ gives the opposite direction of travel and so the opposite $\hat{B}$. Read ω and k straight off the argument: the speed is ω/k. If both fields carry a common vector such as $(2\hat{i} + \hat{j})$, its magnitude is $\sqrt{5}$ times the coefficient.

SANITY CHECK

$B_0 = E_0/c$ is tiny: a 3 V/m wave has $B_0 = 10$ nT. Both fields still carry equal energy, because B is multiplied by c in the energy.

3 Energy, intensity, momentum and radiation pressure

ENERGY OF A WAVE The fields store energy, $\frac{1}{2}\varepsilon_0 E^2$ in the electric part and $\dfrac{B^2}{2\mu_0}$ in the magnetic part. For a plane wave these are **equal at every instant**. The energy moves with the wave at speed c; the intensity I (W/m^2) is the average energy flowing through unit area per second.

ENERGY DENSITY

$$u = \varepsilon_0 E^2 = \frac{B^2}{\mu_0} \text{ (instantaneous)}$$

$$u_{avg} = \tfrac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

half in the electric field, half in the magnetic field

INTENSITY

$$I = u_{avg}\, c = \tfrac{1}{2}\varepsilon_0 c E_0^2 = \varepsilon_0 c E_{rms}^2$$

from intensity: $E_0 = \sqrt{\dfrac{2I}{\varepsilon_0 c}}$; 1 W/m^2 gives 27 V/m

POINT SOURCE

$$I = \frac{P}{4\pi r^2}$$

$I \propto 1/r^2$ and $E_0 \propto 1/r$ for an isotropic source

MOMENTUM

$$p = \frac{U}{c} \text{ (absorbed)}$$

$$p = \frac{2U}{c} \text{ (reflected)}$$

a wave carrying energy U also carries momentum; reflection reverses it

RADIATION PRESSURE AT NORMAL INCIDENCE

SURFACE	PRESSURE	FORCE ON AREA A
Perfect absorber	I/c	IA/c
Perfect reflector	$2I/c$	$2IA/c$
Reflects a fraction r	$(1+r)I/c$	$(1+r)IA/c$

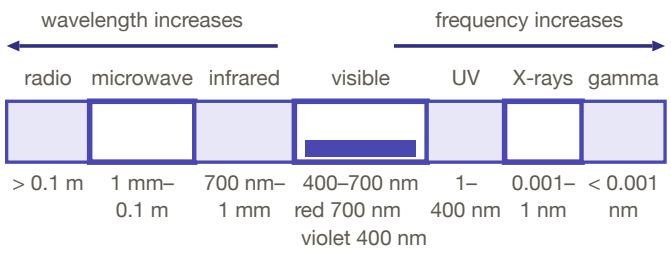
WHY IT WORKS

A beam that is absorbed hands over its momentum U/c; one that bounces back reverses it, so the surface receives twice as much, $2U/c$. That is why a reflecting sail feels double the pressure of a black one. Radiation pressure is tiny, a few 10^{-6} Pa even for full sunlight, because c is large.

TRAP

Do not mix peak and RMS: $I = \frac{1}{2}\varepsilon_0 c E_0^2$ uses the **peak**, $I = \varepsilon_0 c E_{rms}^2$ uses the RMS. A perfect absorber gets I/c, not $2I/c$. Convert W/cm^2 and cm^2 to SI before using the formulas.

4 The electromagnetic spectrum



Frequency rises and wavelength falls from left to right; visible light is a narrow band in the middle

All electromagnetic waves travel at c in vacuum; they differ only in frequency and wavelength, and so in how they are produced, detected and used.

The wavelength ranges are approximate and overlap at their edges; the order is what matters.

THE BANDS AND WHAT THEY DO

BAND	WAVELENGTH (APPROX.)	PRODUCED BY	USED FOR
Radio	above 0.1 m	accelerated charges in antenna circuits	radio, TV, mobile phones
Microwave	1 mm to 0.1 m	klystron, magnetron, Gunn diode	radar, microwave ovens, satellite links
Infrared	700 nm to 1 mm	hot bodies, molecular vibrations	remote controls, thermal imaging, physiotherapy
Visible	400 to 700 nm	electron transitions in atoms, hot bodies	vision, photography, photocells
Ultraviolet	1 to 400 nm	the Sun, arcs, special lamps	sterilising water, LASIK; the ozone layer absorbs most of it
X-rays	10^{-3} to 1 nm	high-energy electrons striking a metal target	medical imaging, security scanners
Gamma rays	below 10^{-3} nm	nuclear decay	radiotherapy, sterilisation

KEY IDEA

Frequency order (low to high): **R**adio, **M**icrowave, **I**nfrared, **V**isible, **U**ltraviolet, **X**-ray, **G**amma. Wavelength runs the other way. Ordering questions ask for one or the other, so read whether "ascending" refers to wavelength, frequency or energy.

CONNECTS TO

Photon energy $E = hf = \dfrac{hc}{\lambda}$: with $hc = 1240$ eV nm, 500 nm light has 2.5 eV. Higher-frequency bands carry more energy per photon; see **Dual Nature**.

TWO APPLICATIONS

Greenhouse effect: the Earth emits **infrared**, which water vapour and carbon dioxide absorb and re-radiate back; visible light passes in. A microwave oven works because water molecules absorb microwaves; the oven walls reflect them.

SYLLABUS NOTE

The ranges differ slightly between textbooks; the ordering is what is examined.

5 Worked examples

B FROM E

A wave has $E_0 = 3$ V/m. Find B_0.

$B_0 = E_0/c = 3/(3 \times 10^8) = 10^{-8}$ T $= 10$ nT

SPEED FROM THE ARGUMENT

$E = E_0 \sin(5 \times 10^6 z - 1.5 \times 10^{15} t)$. Find speed, frequency, wavelength, and the band.

$v = \omega/k = 3 \times 10^8$ m/s; $f = \omega/2\pi = 2.4 \times 10^{14}$ Hz; $\lambda = 2\pi/k = 1.26$ μm: infrared

DIRECTION OF B

A wave has $\vec{E}$ along $+\hat{i}$ and travels along $+\hat{k}$; then the same $\vec{E}$ with travel along $-\hat{k}$. Direction of $\vec{B}$ each time?

$+\hat{k}$: $\hat{i}\times\hat{j}=\hat{k}$, so $\vec{B}$ along $+\hat{j}$; $-\hat{k}$: need $\hat{i}\times\hat{B}=-\hat{k}$, so $\vec{B}$ along $-\hat{j}$

INTENSITY

The peak electric field of a wave is 200 V/m. Its intensity?

$I=\frac{1}{2}\varepsilon_0 cE_0^2=\frac{1}{2}\times 2.65\times10^{-3}\times4\times10^{4}\approx 53$ W/m^2

ISOTROPIC SOURCE

A 1000 W source radiates uniformly. Intensity and peak electric field 10 m away?

$I=\dfrac{1000}{4\pi\times100}=0.80$ W/m^2; $E_0=\sqrt{\dfrac{2I}{\varepsilon_0 c}}\approx 24$ V/m

FORCE ON A PANEL

50 W/m^2 falls normally on a $1\ \text{m}^2$ panel that reflects 25% and absorbs the rest. Force?

$F=\dfrac{(1+r)IA}{c}=\dfrac{1.25\times50}{3\times10^8}\approx 2.1\times10^{-7}$ N

MOMENTUM

A surface absorbs 6.48×10^5 J of radiation. Momentum delivered?

$p=U/c=\dfrac{6.48\times10^5}{3\times10^8}=2.16\times10^{-3}$ kg m/s

IN A MEDIUM

Light of 600 nm (vacuum) enters glass with $\varepsilon_r=2.25$, $\mu_r=1$. Speed, wavelength, frequency?

$n=1.5$: $v=2\times10^8$ m/s, $\lambda=400$ nm, frequency unchanged (5×10^{14} Hz)

ORDER THE BANDS

Arrange microwave, ultraviolet, infrared and X-rays in ascending wavelength.

X-rays, ultraviolet, infrared, microwave

DISPLACEMENT CURRENT

The field between the plates of a capacitor of area A changes at rate dE/dt. Find I_d.

$I_d=\varepsilon_0\dfrac{d\Phi_E}{dt}=\varepsilon_0 A\dfrac{dE}{dt}$, equal to the wire current

6 Last-minute checklist

BEFORE YOU ANSWER

Peak or RMS? Which way does the wave travel (sign in $kz\pm\omega t$)? Is the question asking for wavelength, frequency or energy order? Are the units SI?

- $I_d=\varepsilon_0\,d\Phi_E/dt=C\,dV/dt$; $I_{d,max}=V_0\omega C$; zero for a steady field; equals the wire current in the gap.
- $c=1/\sqrt{\mu_0\varepsilon_0}=f\lambda=\omega/k$; $v=c/n$; frequency stays the same in a medium.
- $E=cB$, in phase; travel along $\vec{E}\times\vec{B}$; transverse; no deflection by fields; no medium needed.
- $u_{avg}=\frac{1}{2}\varepsilon_0E_0^2$, shared equally; $I=\frac{1}{2}\varepsilon_0cE_0^2=\varepsilon_0cE_{rms}^2$; point source $P/4\pi r^2$.
- $p=U/c$ (absorbed), $2U/c$ (reflected); pressure I/c, $2I/c$, $(1+r)I/c$.
- Spectrum by frequency: radio, microwave, infrared, visible, UV, X-ray, gamma; wavelength is the reverse.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**