JEE MAIN · PAPER 1 · PHYSICS

14 Electromagnetic Induction & AC

Formula & concept sheet

A changing magnetic flux makes an emf: this chapter follows that idea from a single loop to motional emf, inductors and LR circuits, then to alternating current in R, L and C, resonance, the generator and the transformer. Each idea is explained first, then the formulas, shortcuts and traps.

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READ ALONGSIDE NCERT XII · Electromagnetic Induction; Alternating Current

HOW TO USE THIS SHEET

1. **Skim** — Read the Key ideas, then the formula cards.
2. **Memorise** — Learn the formulas and check every Trap box.
3. **Test** — Do the worked examples, then this chapter's PYQs.
4. **Revise** — Use the last-minute checklist before the exam.

Formula memorise · **Key idea** why it works · **Trap** marks lost here · **PYQ** how it is asked

Practise this chapter with real past-paper questions on Prep Vault. Written for revision, not as a first read.

CHAPTER 14

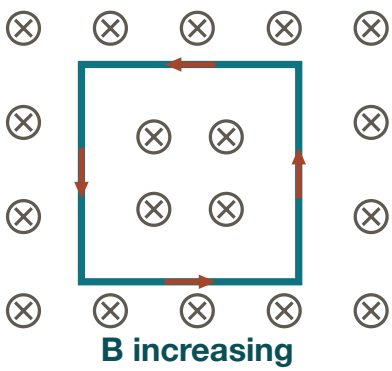
Electromagnetic Induction & AC

NCERT XII · Electromagnetic Induction; Alternating Current

Covers: Faraday's and Lenz's laws · motional emf · eddy currents · self and mutual inductance · LR circuit · peak and RMS values · reactance and impedance · series LCR and resonance · power and power factor · LC oscillations · AC generator and transformer.

1 Magnetic flux, Faraday's law and Lenz's law

FARADAY'S LAW An emf is induced in a loop whenever the magnetic flux through it changes. Its size is the rate of change of flux, $\varepsilon = -N\dfrac{d\Phi}{dt}$, where the flux through one turn is $\Phi = BA\cos\theta$ (weber, Wb) and θ is the angle between $\vec{B}$ and the **normal** to the loop.



B rises into the page, so the induced current runs anticlockwise and its field points out of the page (Lenz)

- Only the **change** of flux matters, not its value. Flux can change because B changes, the area changes, or the angle θ changes.
- **Lenz's law** (the minus sign): the induced current flows so that its own field **opposes the change** in flux, not the flux itself. This is energy conservation: if it helped the change, the effect would feed itself without limit.
- The induced electric field circulates round the loop; unlike an electrostatic field it is **not conservative**, $\oint \vec{E}\cdot d\vec{l} = -d\Phi/dt$.

FLUX

$\Phi = BA\cos\theta$

θ from the normal; given the angle with the **plane**, use $\sin$ instead

AVERAGE EMF

$\varepsilon_{avg} = N\dfrac{|\Delta\Phi|}{\Delta t}$

for a loop pulled out or rotated: $\Delta\Phi = BA(\cos\theta_1 - \cos\theta_2)$

INDUCED CHARGE

$q = \dfrac{N|\Delta\Phi|}{R}$

independent of how fast the flux changes; R = total resistance

CHANGING FIELD, FIXED LOOP

$\varepsilon = A\dfrac{dB}{dt}$ (one turn, $B \perp$ loop)

if $\Phi(t)$ is given, differentiate: $I = \dfrac{1}{R}\left|\dfrac{d\Phi}{dt}\right|$

ROTATING COIL

$\Phi = NBA\cos\omega t,\ \ \varepsilon = NBA\omega\sin\omega t$

peak emf $\varepsilon_0 = NBA\omega$; this is the AC generator

TRAP

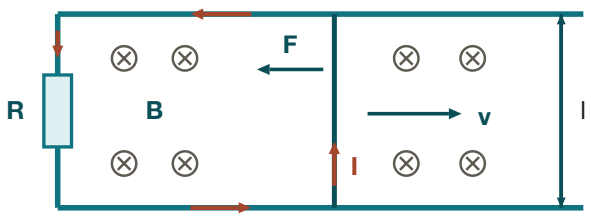
The angle in $\Phi = BA\cos\theta$ is with the normal. Lenz's law opposes the **change**: if the flux through a loop is decreasing, the induced field points the same way as the original.

QUICK CHECK

$N = 100$ turns, the flux changes by 0.02 Wb, total resistance $10\ \Omega$. Charge that flows?

Answer: $q = \dfrac{N\Delta\Phi}{R} = 0.2$ C, however long it takes.

2 Motional emf and eddy currents



ε = Blv

Positive charges in the rod feel $q\vec{v} \times \vec{B}$ upward, so I runs up the rod; the force $I\vec{l} \times \vec{B}$ on the rod opposes v

MOTIONAL EMF A conductor of length l moving at speed v across a field B (all three mutually perpendicular) has an emf $\varepsilon = Blv$. It is Faraday's law in another form: the area swept per second is lv, so the flux through the circuit changes at the rate Blv.

FROM THE LORENTZ FORCE

$F = qvB$ along the rod

each charge is pushed to one end

$qE = qvB \;\Rightarrow\; E = vB$

charge piles up until the electric force balances

$\varepsilon = El = Blv$

potential difference across the rod

ROD ON RAILS

$I = \dfrac{Blv}{R}, \quad F = BIl = \dfrac{B^2l^2v}{R}$

the magnetic force opposes the motion (Lenz); power $Fv = I^2R$

TERMINAL SPEED

$v_t = \dfrac{FR}{B^2l^2}$

when an applied force F balances the magnetic drag

ROTATING ROD OR DISC

$\varepsilon = \frac{1}{2}B\omega l^2$

about one end, or from centre to rim of a disc of radius l; the $\frac{1}{2}$ is the average speed along the rod

EDDY CURRENTS

- When a bulk conductor (a metal plate, a core) is in a changing flux, currents circulate inside it and, by Lenz's law, **oppose** the change. They heat the metal.
- Used for **magnetic braking** and damping (a galvanometer coil on a metal frame, induction furnaces); reduced in transformer and motor cores by **laminating** them, thin insulated sheets that cut the current paths.

WHY IT WORKS

Pulling a metal plate out of a magnet is hard because the eddy currents make a force opposing the motion. The work you do is turned into heat, so energy is conserved.

TRAP

$\varepsilon = Blv$ needs $\vec{B}$, $\vec{v}$ and the rod mutually perpendicular; otherwise take only the perpendicular components, $\varepsilon = (\vec{v} \times \vec{B}) \cdot \vec{l}$. In the Earth's field a horizontal wire uses the horizontal component B_H when it falls and the vertical component B_V when it moves sideways.

3 Self and mutual inductance

INDUCTANCE A changing current in a coil changes its own flux, and so induces an emf in the same coil: $\varepsilon = -L\dfrac{dI}{dt}$. L (henry) depends only on geometry and the core material. **Mutual inductance** M does the same between two coils: $\varepsilon_2 = -M\dfrac{dI_1}{dt}$, with $M_{12} = M_{21}$.

DEFINITION

$N\Phi = LI, \;\; N_2\Phi_2 = MI_1$

flux linked with the coil per unit current

LONG SOLENOID

$L = \mu_0 n^2 A l = \dfrac{\mu_0 N^2 A}{l}$

$n = N/l$; multiply by μ_r with a core; fixed N and A gives $L \propto 1/l$

ENERGY AND DENSITY

$U = \frac{1}{2} L I^2$

$u = \dfrac{B^2}{2\mu_0}$

energy stored in the magnetic field of the coil

COAXIAL SOLENOIDS

$M = \mu_0 n_1 n_2 \pi r_1^2 \, l$

r_1 the radius of the inner one; equals $\dfrac{\mu_0 N_1 N_2 A_1}{l}$

SMALL LOOP IN A LARGE LOOP

$M = \dfrac{\mu_0 a}{2R}$

small loop of area a at the centre of a large circular loop of radius R (field at the centre $\mu_0 I/2R$)

COUPLING

$M = k\sqrt{L_1 L_2}, \;\; 0 \le k \le 1$

$k = 1$ when all the flux of one coil links the other

INDUCTORS IN SERIES OR PARALLEL

series: $L = L_1 + L_2$

parallel: $\dfrac{1}{L} = \dfrac{1}{L_1} + \dfrac{1}{L_2}$

when the coils are not coupled

WHY IT WORKS

An inductor opposes **changes** of current, as mass opposes changes of velocity: L is the electrical inertia. That is why the current in a coil cannot jump, and why $\frac{1}{2} L I^2$ plays the part of $\frac{1}{2} m v^2$.

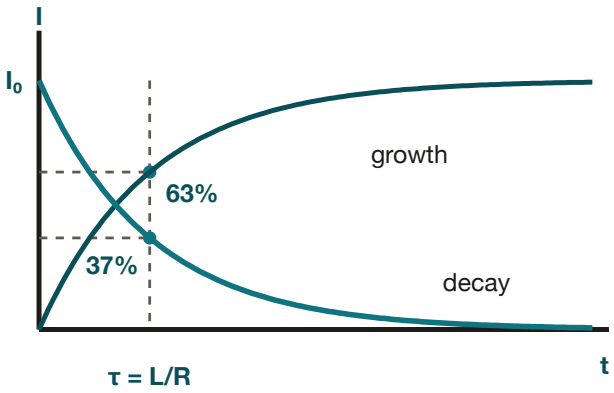
TRAP

M and L are fixed by geometry; never use $M > \sqrt{L_1 L_2}$. Convert mH and μF to SI before using them.

CONNECTS TO

The energy density $\dfrac{B^2}{2\mu_0}$ is the magnetic partner of $\frac{1}{2}\varepsilon_0 E^2$ in **Electrostatics**.

4 The LR circuit



After one time constant $\tau = L/R$ the current has grown to $0.63\, I_0$ or decayed to $0.37\, I_0$

GROWTH AND DECAY The current in an inductor cannot change instantly. Connected to a battery through a resistor it grows to E/R; shorted through the resistor it dies away. Both follow an exponential with the time constant $\tau = L/R$.

R is the **total** resistance in the loop, including the battery's internal resistance.

GROWTH (BATTERY E)

$$I = \frac{E}{R}\left(1 - e^{-t/\tau}\right)$$

$V_L = Ee^{-t/\tau}$; after τ the current is 63% of E/R

DECAY

$$I = I_0 e^{-t/\tau}$$

after τ the current is 37% of I_0

TIME TO REACH A FRACTION f

$$t = \tau \ln \frac{1}{1-f}$$

80% takes $\tau \ln 5 \approx 1.6\,\tau$

INDUCTOR AGAINST CAPACITOR AT THE TWO EXTREME INSTANTS

ELEMENT	JUST AFTER SWITCHING ON	LONG AFTER (STEADY STATE)
Inductor	acts as a gap: no current yet	acts as a plain wire (only its own resistance)
Capacitor	acts as a wire: uncharged	acts as a gap: no current

KEY IDEA

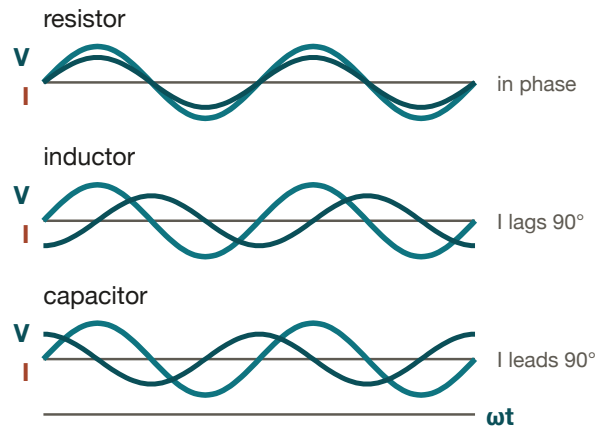
For the current just after a switch is closed, replace each inductor by an open gap and each uncharged capacitor by a wire. For a long time afterwards, do the opposite.

TRAP

Leave out the internal resistance of the battery and τ and the final current are both wrong. Opening a circuit with a large L suddenly gives a huge emf $L\,dI/dt$, which is why a spark appears at the switch.

5 Alternating current: R, L and C alone

ALTERNATING VOLTAGE A sinusoidal source $V = V_0 \sin \omega t$ with $\omega = 2\pi f$. Its mean over a full cycle is zero, so we use the **RMS** value, the steady value that would give the same heating: $V_{rms} = \frac{V_0}{\sqrt{2}}$, $I_{rms} = \frac{I_0}{\sqrt{2}}$. Meters and the "220 V" of the mains are RMS (peak about 311 V).



Same time axis: I in phase with V for R, lagging by 90° for L and leading by 90° for C

- **Resistor**: V and I in phase, $I_0 = V_0/R$.
- **Inductor**: I **lags** V by 90°; reactance $X_L = \omega L$ (grows with frequency; blocks high frequencies).
- **Capacitor**: I **leads** V by 90°; reactance $X_C = \frac{1}{\omega C}$ (falls with frequency; blocks DC).
- Pure L or pure C: average power is **zero** (the energy only shuttles in and out).

RMS AND PEAK

$$V_{rms} = \frac{V_0}{\sqrt{2}} \approx 0.707\,V_0$$

sinusoidal waveform only; same for current

MEAN OVER A HALF CYCLE

$$\frac{2I_0}{\pi} \approx 0.637\,I_0$$

the mean over a full cycle is 0

REACTANCE

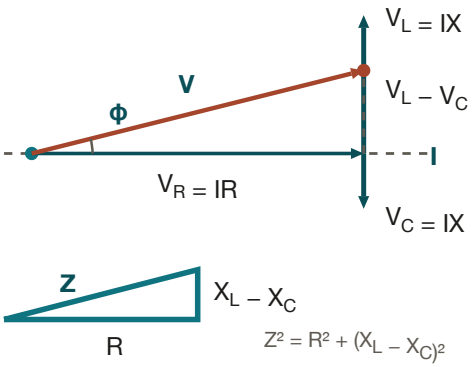
$$X_L = \omega L, \quad X_C = \frac{1}{\omega C}$$

ohm; $\omega = 2\pi f$, and $\omega = 100\pi$ for 50 Hz mains

REMEMBER

"CIVIL": in a Capacitor the current I leads V; in an inductor L, V leads I. Mains: $\omega = 100\pi \approx 314$ rad/s, so π in a given L or C usually cancels.

6 Series LCR circuit, power and power factor



V_L leads I by 90° and V_C lags it; the net reactive $V_L - V_C$ sets $\tan\varphi = (X_L - X_C)/R$

PHASORS

- The same current I flows through all three elements. V_R is along I, V_L leads it by 90°, V_C lags it by 90°.
- The voltages add as **vectors**, not numbers: $V = \sqrt{V_R^2 + (V_L - V_C)^2}$.
- Divide by I for the impedance triangle: sides R and $X_L - X_C$, hypotenuse Z.

IMPEDANCE

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$I_{rms} = \frac{V_{rms}}{Z}$; only RL: $\sqrt{R^2 + \omega^2 L^2}$; only RC: $\sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

PHASE ANGLE

$$\tan\phi = \frac{X_L - X_C}{R}$$

$\phi > 0$: inductive, current lags; $\phi < 0$: capacitive, current leads

AVERAGE POWER

$$P = V_{rms} I_{rms} \cos\phi = I_{rms}^2 R$$

power factor $\cos\phi = \frac{R}{Z}$; given $V(t)$ and $I(t)$: $P = \frac{1}{2} V_0 I_0 \cos\phi$

WATTLESS CURRENT

$I_{rms} \sin\phi$

the component of the current that is 90° out of phase with V; it carries no average power

APPARENT POWER

$$S = V_{rms} I_{rms}, \quad P = S\cos\phi$$

volt-ampere; $\cos\phi = 1$ for a pure resistance, 0 for a pure L or C

SERIES LCR NUMBERS

1. Find $X_L = \omega L$ and $X_C = 1/\omega C$ in ohms first (check for $X_L = X_C$: then $Z = R$ and $\cos\phi = 1$).
2. $Z = \sqrt{R^2 + (X_L - X_C)^2}$, then $I = V/Z$ and $\cos\phi = R/Z$.
3. $P = I^2 R$ (or $VI\cos\phi$). A 3-4-5 triangle gives $\cos\phi = 0.6$ at once.

TRAP

Average power is $\frac{1}{2} V_0 I_0 \cos\phi$ or $V_{rms} I_{rms} \cos\phi$, never $V_0 I_0$. Do not add V_R, V_L, V_C as numbers: V_L and V_C point in opposite directions.

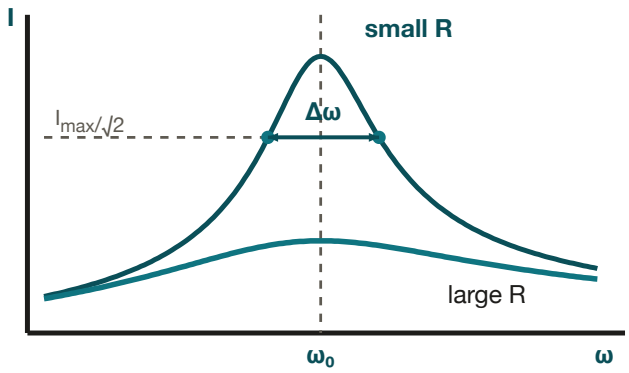
QUICK CHECK

$V = V_0 \sin\omega t$ with $V_0 = 100$ V, $I = 0.1\sin(\omega t + 60^\circ)$ A. Average power?

Answer: $P = \frac{1}{2} \times 100 \times 0.1 \times \cos 60^\circ = 2.5$ W.

7 Resonance and LC oscillations

RESONANCE When $X_L = X_C$ the two reactances cancel, the impedance is smallest ($Z = R$), the current is largest and in phase with the source ($\phi = 0$). It happens at $\omega_0 = \dfrac{1}{\sqrt{LC}}$.



A small R gives a tall, sharp peak at ω_0; the half-power width is $\Delta\omega = R/L$

FEATURES

- As ω rises from zero the current climbs to its peak at ω_0 and then falls again.
- At resonance $V_L = V_C = QV$: each can be far larger than the supply voltage, though they cancel in the loop.
- A small R gives a tall, sharp peak (high Q); a large R a low, broad one.

SERIES LCR AGAINST FREQUENCY

FREQUENCY	REACTANCES	CIRCUIT BEHAVES AS	CURRENT
$\omega < \omega_0$	$X_C > X_L$	capacitive	leads V ($\phi < 0$)
$\omega = \omega_0$	$X_L = X_C$	purely resistive	in phase with V, at its maximum
$\omega > \omega_0$	$X_L > X_C$	inductive	lags V ($\phi > 0$)

RESONANT FREQUENCY

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$Z = R,\ I_{max} = V/R,\ \cos\phi = 1$

QUALITY FACTOR

$$Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_0}{\Delta\omega}$$

high Q means a sharp resonance

BANDWIDTH

$$\Delta\omega = \frac{R}{L}$$

half-power points at about $\omega_0 \pm \dfrac{R}{2L}$ when Q is large

LC OSCILLATIONS (NO RESISTANCE)

$$\omega = \frac{1}{\sqrt{LC}}, \quad q = Q_0 \cos\omega t$$

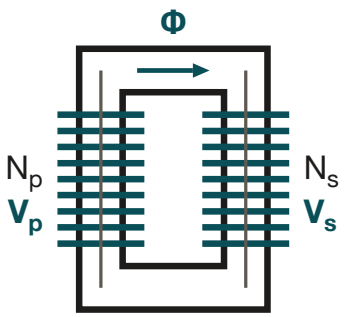
$$I_{max} = \frac{Q_0}{\sqrt{LC}} = V_0\sqrt{\frac{C}{L}}$$

from $\dfrac{Q_0^2}{2C} = \tfrac{1}{2}LI_{max}^2$: the energy moves between the capacitor and the inductor, total constant

CONNECTS TO

LC oscillations are SHM: $L \leftrightarrow m$, $\dfrac{1}{C} \leftrightarrow k$, $q \leftrightarrow x$, $I \leftrightarrow v$, so $\omega = \sqrt{k/m}$ becomes $\dfrac{1}{\sqrt{LC}}$. See **Oscillations**.

8 AC generator, transformer and power transmission



Vs/Vp = Ns/Np

tep-up: more turns on the secondar

An alternating flux Φ in the core links both coils, so each turn gives the same emf: $V_s/V_p = N_s/N_p$ (ideal)

- **Generator**: a coil of N turns rotating at ω in a uniform field has flux $NBA\cos\omega t$, so $\varepsilon = NBA\omega\sin\omega t$.
- **Transformer**: an alternating current in the primary makes an alternating flux in the iron core, which links the secondary. Each turn has the same emf, so the voltages are in the ratio of the turns. It does **not** work on DC, since a steady current gives no changing flux.
- Losses: resistance of the windings, eddy currents (laminated core), hysteresis, flux leakage.

IDEAL TRANSFORMER

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$

step-up if $N_s > N_p$; power in = power out

EFFICIENCY

$$\eta = \frac{V_s I_s}{V_p I_p}$$

apply it to the power, not to the voltage

LINE LOSS

$$P_{loss} = I^2 R_{line} = \left(\frac{P}{V}\right)^2 R_{line}$$

sending the same power at a higher voltage lowers I and cuts the loss as $1/V^2$

KEY IDEA

A step-up transformer raises the voltage and lowers the current by the same factor, so the power is unchanged. Power is sent at high voltage so the heating loss I^2R in the lines is small.

TRAP

An ideal transformer conserves **power**, not voltage. Chain: turns ratio gives the secondary voltage, power conservation gives the current, and efficiency multiplies the output power only.

9 Worked examples

HIDDEN RESONANCE

A series circuit has $L = \dfrac{100}{\pi}$ mH, $C = \dfrac{10^{-3}}{\pi}$ F, $R = 10\ \Omega$ on 220 V, 50 Hz. Power factor?

$X_L = 100\pi \times 0.1/\pi = 10\ \Omega$ and $X_C = 1/(100\pi \times 10^{-3}/\pi) = 10\ \Omega$, so $Z = R$ and $\cos\phi = 1$

LC PEAK CURRENT

A $100\ \mu$F capacitor charged to 12 V is joined to 6.4 mH. Maximum current?

$I_{max} = V_0\sqrt{C/L} = 12 \times \sqrt{0.0156} = 12 \times 0.125 = 1.5$ A

CURRENT FROM FLUX

The flux through a coil is $\Phi = 5t^2 - 36t + 1$ (Wb) and the resistance is $8\ \Omega$. Current at $t = 2$ s?

$\varepsilon = \dfrac{d\Phi}{dt} = 10t - 36 = -16$ V; $|I| = 16/8 = 2$ A

TRANSFORMER

Primary 230 V, turns ratio $10:1$ (step-down), load $46\ \Omega$. Power in the load?

$V_s = 23$ V; $P = V_s^2/R = 529/46 = 11.5$ W

ROD ON RAILS

A rod $l = 1$ m moves at $v = 5$ m/s with $B = 0.2$ T on rails closed by $2\ \Omega$. Emf, current, drag?

$\varepsilon = Blv = 1$ V; $I = 0.5$ A; $F = BIl = 0.1$ N, against the motion

ROTATING DISC

A disc of radius 0.5 m turns at 10 rad/s in $B = 0.4$ T (along the axis). Emf between centre and rim?

$\frac{1}{2}B\omega r^2 = \frac{1}{2} \times 0.4 \times 10 \times 0.25 = 0.5$ V

LR CURRENT

$L = 2$ H, $R = 10\ \Omega$, $E = 20$ V (no internal resistance). Time constant and the current at $t = \tau$?

$\tau = L/R = 0.2$ s; $I = 2\left(1 - e^{-1}\right) \approx 1.26$ A

Q AND BANDWIDTH

$L = 0.5$ H, $C = 2\ \mu$F, $R = 10\ \Omega$. Find Q and $\Delta\omega$.

$\omega_0 = 1/\sqrt{LC} = 1000$ rad/s; $Q = \omega_0 L/R = 50$; $\Delta\omega = R/L = 20$ rad/s

POWER FACTOR

A series circuit has $R = 3\ \Omega$ and net reactance $4\ \Omega$ on 100 V (RMS). Power factor and power?

$Z = 5\ \Omega$, $\cos\phi = 0.6$; $I = 20$ A; $P = I^2R = 1200$ W $(= VI\cos\phi)$

TRANSMISSION LOSS

1 MW is sent over a line of $10\ \Omega$ at 11 kV, and then at 110 kV. Loss in each case?

$P_{loss} = (P/V)^2 R$: about 83 kW at 11 kV, about 0.83 kW at 110 kV

10 Last-minute checklist

BEFORE YOU ANSWER

Is the angle with the normal or the plane? Is the quantity RMS or peak? Does the circuit sit at resonance ($X_L = X_C$)? Are mH and μF in SI?

- $\varepsilon = -N\,d\Phi/dt$; $\Phi = BA\cos\theta$; $q = N\Delta\Phi/R$; Lenz: oppose the **change**.
- Motional: Blv; rotating rod or disc $\frac{1}{2}B\omega l^2$; rod on rails $F = B^2l^2v/R$, $v_t = FR/B^2l^2$.
- $L = \mu_0 N^2 A/l$; $U = \frac{1}{2}LI^2$; LR: $\tau = L/R$, growth $I = \dfrac{E}{R}(1 - e^{-t/\tau})$; inductor is a gap at once, a wire later.
- $V_{rms} = V_0/\sqrt{2}$; $X_L = \omega L$ (current lags), $X_C = 1/\omega C$ (current leads); $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
- $P = V_{rms}I_{rms}\cos\phi = I^2R$, $\cos\phi = R/Z$; pure L or C: $P = 0$; $P = \frac{1}{2}V_0 I_0 \cos\phi$ from $V(t), I(t)$.
- Resonance $\omega_0 = 1/\sqrt{LC}$, $Z = R$; $Q = \omega_0 L/R$, $\Delta\omega = R/L$; LC peak current $V_0\sqrt{C/L}$.
- Transformer $V_s/V_p = N_s/N_p = I_p/I_s$ (ideal); line loss $(P/V)^2R$; no transformer action with DC.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**