JEE MAIN · PAPER 1 · PHYSICS

13 Magnetic Effects of Current & Magnetism

Formula & concept sheet

Moving charges make magnetic fields and feel magnetic forces. The sheet builds the field of wires, loops and solenoids, follows charges and currents in a field, explains the moving-coil galvanometer, and ends with bar magnets and the three kinds of magnetic material. Each idea is explained first, then the formulas, shortcuts and traps.

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READ ALONGSIDE NCERT XII · Moving Charges and Magnetism; Magnetism and Matter

HOW TO USE THIS SHEET

1. **Skim** — Read the Key ideas, then
2. **Memorise** — Learn the formulas and
3. **Test** — Do the worked examples,
4. **Revise** — Use the last-minute

CHAPTER 13

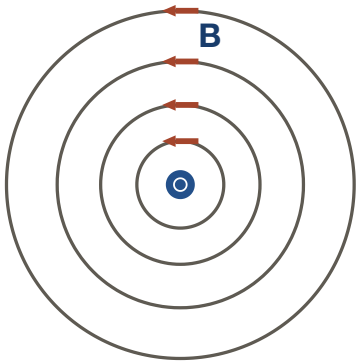
Magnetic Effects of Current & Magnetism

NCERT XII · Moving Charges and Magnetism; Magnetism and Matter

Covers: Biot-Savart law · Ampere's law, solenoid, toroid · force on a moving charge · cyclotron · force on a current · parallel wires · torque on a loop · moving-coil galvanometer · bar magnet as a dipole · dia-, para- and ferromagnetism.

1 Magnetic field of a current: Biot-Savart law

BIOT-SAVART LAW Every small element $I\,d\vec{l}$ of a current makes a field $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\,d\vec{l}\times\hat{r}}{r^2}$ at a point a vector $\vec{r}$ away. The field of a whole wire is the sum (integral) over its elements. Here $\mu_0 = 4\pi\times10^{-7}$ T m/A, so $\dfrac{\mu_0}{4\pi} = 10^{-7}$ T m/A.



I out of page

$B = \dfrac{\mu_0 I}{2\pi r}$: circles around the wire, anticlockwise for current out of the page

- **Direction.** Right-hand thumb rule: thumb along the current in a straight wire, fingers curl the way $\vec{B}$ circulates. For a loop, curl the fingers along the current and the thumb gives $\vec{B}$ through the loop.
- dB is zero along the line of the element ($\sin\theta = 0$): a straight lead that points at the observation point adds nothing.
- Field strength falls as $1/r^2$ for an element, so a long straight wire gives only $1/r$ because many elements add up.

FINITE STRAIGHT WIRE

$$B = \frac{\mu_0 I}{4\pi d}(\sin\phi_1 + \sin\phi_2)$$

d = perpendicular distance; ϕ_1, ϕ_2 = angles from that perpendicular to the two ends

INFINITE WIRE

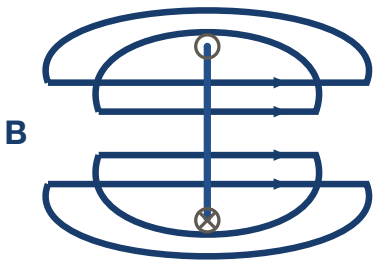
$$B = \frac{\mu_0 I}{2\pi d}$$

both angles 90°; a semi-infinite wire, at a point level with its end, gives half: $\dfrac{\mu_0 I}{4\pi d}$

CIRCULAR ARC, AT THE CENTRE

$$B = \frac{\mu_0 I\theta}{4\pi R}$$

θ in radians; full loop of N turns $\dfrac{\mu_0 N I}{2R}$, semicircle $\dfrac{\mu_0 I}{4R}$



B is along the axis inside the loop, dense there and weak outside

LOOP ON ITS AXIS

$$B = \frac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}}$$

$x = 0$ gives the centre value; $x \gg R$ gives $\frac{\mu_0}{2\pi}\frac{NI\pi R^2}{x^3}$, the dipole field with moment $NI\pi R^2$

CENTRE OF COMMON SHAPES (CURRENT I)

SHAPE	FIELD
Semicircle, radius R	$\mu_0 I/4R$
Quarter circle	$\mu_0 I/8R$
Square, side a	$2\sqrt{2}\,\mu_0 I/\pi a$
Equilateral triangle, side a	$9\mu_0 I/2\pi a$

FINITE WIRE FROM BIOT-SAVART

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl\,\sin\theta}{r^2}$$

element at distance r, angle θ to the line from the element to the point

$$l = d\tan\phi,\; r = \frac{d}{\cos\phi},\; \sin\theta = \cos\phi \;\Rightarrow\; dB = \frac{\mu_0 I}{4\pi d}\cos\phi\, d\phi$$

change the variable to the angle ϕ from the perpendicular

$$B = \frac{\mu_0 I}{4\pi d}\int_{-\phi_1}^{\phi_2}\cos\phi\, d\phi = \frac{\mu_0 I}{4\pi d}(\sin\phi_1 + \sin\phi_2)$$

infinite wire: $\phi_1 = \phi_2 = 90^\circ$

COMPOSITE WIRE SHAPES

1. Split the wire into straight pieces and arcs. A piece whose line passes through the point gives zero.
2. Use $\frac{\mu_0 I}{4\pi d}(\sin\phi_1 + \sin\phi_2)$ for a straight piece and $\frac{\mu_0 I\theta}{4\pi R}$ for an arc.
3. Give each piece its direction (into or out of the page, by the thumb rule) and add with signs.

TRAP

Do not count a segment that lies along the line through the point: its field there is zero. Also, $\mu_0/4\pi$ is 10^{-7}, not $4\pi \times 10^{-7}$.

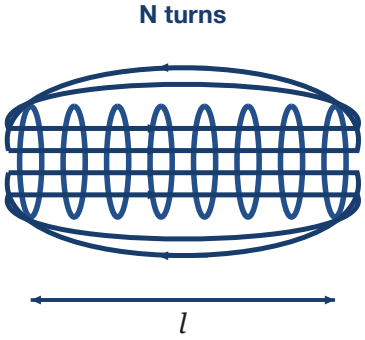
QUICK CHECK

A wire carries 2 A along a semicircle of radius 0.1 m, with straight leads that pass through the centre. Field at the centre?

Answer: the leads give zero; the arc gives $\frac{\mu_0 I}{4R} = \frac{4\pi \times 10^{-7} \times 2}{0.4} \approx 6.3 \times 10^{-6}$ T.

2 Ampere's law: wire, solenoid and toroid

AMPERE'S CIRCUITAL LAW The line integral of $\vec{B}$ round any closed loop equals μ_0 times the current passing through the loop, $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$. It is always true; it **finds** B only when symmetry makes B constant (or zero) along the chosen loop.



inside: uniform and dense, $B = \mu_0 n I$; outside: spreading and weak

SOLENOID AND TOROID

- A long solenoid has a uniform field along its axis inside and almost none outside. At the centre of one end the field is half the interior value.
- A toroid is a solenoid bent into a ring: the field is confined to the ring (zero in the hole and outside) and falls as $1/r$ across the turns.

STANDARD RESULTS FROM AMPERE'S LAW

SOURCE	FIELD
Long straight wire, distance r	$B = \dfrac{\mu_0 I}{2\pi r}$
Inside a thick wire of radius a (uniform current), $r < a$	$B = \dfrac{\mu_0 I r}{2\pi a^2}$, rising linearly; equals the outside value at $r = a$
Long solenoid, n turns per metre	$B = \mu_0 n I$ inside; $\frac{1}{2}\mu_0 n I$ at an end; ≈ 0 outside
Toroid, N turns in all, at radius r	$B = \dfrac{\mu_0 N I}{2\pi r}$ inside the ring; 0 outside and in the hole

SOLENOID FROM A RECTANGULAR AMPERIAN LOOP

$$\oint \vec{B} \cdot d\vec{l} = Bl$$

the loop has a side of length l inside, along the axis; the outside side has $B \approx 0$ and the two ends are perpendicular to $\vec{B}$

$$I_{enc} = nlI$$

nl turns cross the loop

$$Bl = \mu_0 nlI \;\Rightarrow\; B = \mu_0 n I$$

independent of the position inside and of the radius

WHY IT WORKS

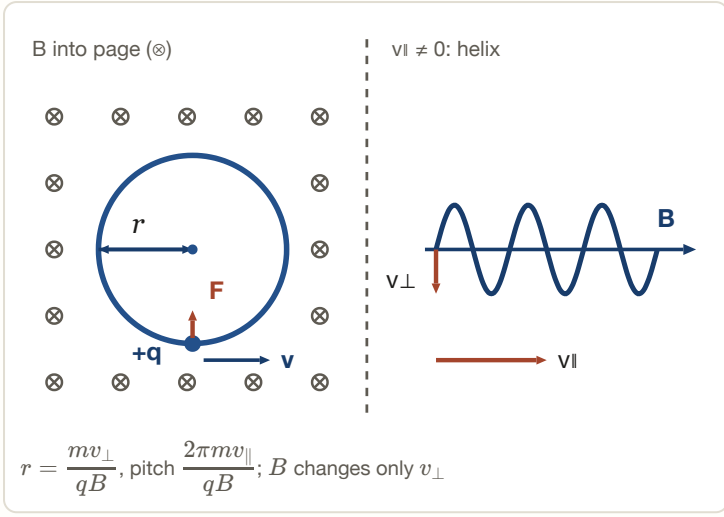
Inside a long solenoid every turn adds to the field along the axis, while outside the fields of the two sides of each turn nearly cancel. The longer the solenoid compared with its radius, the better the uniform-field result holds.

TRAP

Toroid: $B = \mu_0 n I$ with $n = N/2\pi r$ holds **inside the ring only**; outside it, and in the hole, $B = 0$. Solenoid: use n (turns per metre), not the total turns N.

3 A charge in electric and magnetic fields

LORENTZ FORCE A charge q moving with velocity $\vec{v}$ in fields $\vec{E}$ and $\vec{B}$ feels $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. The magnetic part $qvB\sin\theta$ is perpendicular to both $\vec{v}$ and $\vec{B}$, so it bends the path but does **no work**: the speed and the kinetic energy do not change.



$r = \frac{mv_\perp}{qB}$, pitch $\frac{2\pi m v_\parallel}{qB}$; B changes only $v_\perp$

PATHS IN A UNIFORM FIELD

- ✓ $\vec{v} \perp \vec{B}$: a circle. The magnetic force is the centripetal force, $qvB = \frac{mv^2}{r}$.
- ✓ $\vec{v}$ at an angle θ to $\vec{B}$: a helix. $v \sin\theta$ turns the circle, $v \cos\theta$ carries the charge along the field at constant speed.
- ✓ $\vec{v} \parallel \vec{B}$: no force, straight line.
- ✓ The period does not depend on the speed or the radius: faster charges go round a bigger circle in the same time.

RADIUS OF THE CIRCLE

$$r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB}$$

accelerated through V from rest: $r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

PERIOD AND FREQUENCY

$$T = \frac{2\pi m}{qB}, \quad f = \frac{qB}{2\pi m}$$

$\omega = \frac{qB}{m}$, independent of v and r

HELIX

$$\text{pitch} = \frac{2\pi m v \cos\theta}{qB}$$

$$\text{radius} = \frac{mv \sin\theta}{qB}$$

θ between $\vec{v}$ and $\vec{B}$

VELOCITY SELECTOR

$$qE = qvB \Rightarrow v = \frac{E}{B}$$

only the speed matters: charge and mass drop out; $\vec{E}$, $\vec{B}$, $\vec{v}$ mutually perpendicular

FIELD REGION OF WIDTH d

$$\sin\theta = \frac{d}{r}$$

deflection angle on leaving, for $d < r$;

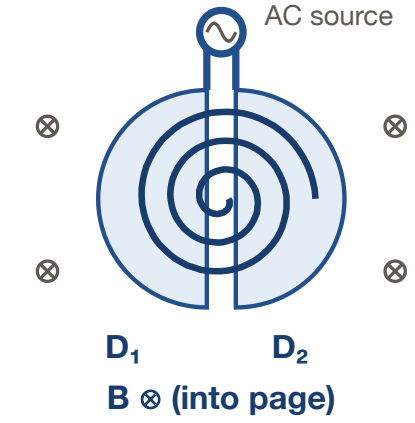
time in the field $t = \frac{\theta m}{qB}$

SAME SPEED, SAME V OR SAME K

same V: $r \propto \sqrt{m/q}$

same K: $r \propto \sqrt{m}/q$

proton : deuteron : α at the same V is $1 : \sqrt{2} : \sqrt{2}$; at the same K it is $1 : \sqrt{2} : 1$



$f = \frac{qB}{2\pi m}$ does not depend on the radius, so the ion spirals outward

CYCLOTRON

- ✓ Two D-shaped dees in a field B with an alternating voltage across the gap. The particle crosses the gap twice a turn and is accelerated each time. Since the orbit time is fixed, a fixed voltage frequency $f = \frac{qB}{2\pi m}$ stays in step as the radius grows.
- ✓ Final energy $K_{max} = \frac{q^2B^2R^2}{2m}$ for outer radius R. It depends on B and R, not on the accelerating voltage; the voltage only sets how many turns it takes.

TRAP

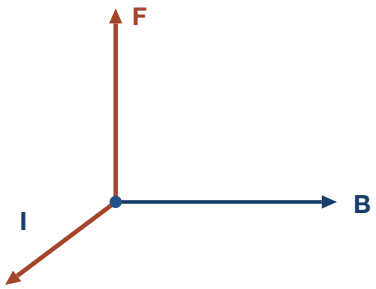
A magnetic field cannot speed a charge up. If a question asks for the work done or the change of kinetic energy in a pure magnetic field, the answer is zero. The force on an electron is opposite to that on a proton for the same $\vec{v}$ and $\vec{B}$.

REMEMBER

Left-hand (Fleming) rule: first finger $\vec{B}$, second finger current (or the motion of a positive charge), thumb the force $\vec{F}$.

4 Force on a current, parallel wires and torque on a loop

FORCE ON A CONDUCTOR A straight wire of length l carrying I in a uniform field feels $\vec{F} = I\,\vec{l} \times \vec{B}$, with magnitude $BIl\sin\theta$. It is the sum of the Lorentz forces on the moving charges inside the wire.



$\vec{F} = I\,\vec{L} \times \vec{B}$: $\vec{F}$ is perpendicular to both the wire and $\vec{B}$

USING IT

- For a wire of any shape in a **uniform** field, use the straight line joining its two ends: $\vec{F} = I\,\vec{L}_{eff} \times \vec{B}$. A semicircle of radius R therefore feels $2IRB$ (perpendicular to the diameter).
- A closed loop in a uniform field has **zero** net force, but it can have a torque.
- Two parallel wires: each sits in the field of the other. Currents in the same direction attract; opposite directions repel.

FORCE ON A WIRE

$F = BIl\sin\theta$

θ between the wire and $\vec{B}$

TWO PARALLEL WIRES

$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$

the ampere: 2×10^{-7} N/m for 1 A in each of two wires 1 m apart

LOOP AS A DIPOLE

$\vec{m} = NI\vec{A}$

A m^2; direction along the normal by the right-hand rule (curl fingers along I)

TORQUE ON A LOOP

$\vec{\tau} = \vec{m} \times \vec{B}$

$\tau = NIAB\sin\theta$

θ between the **normal** and $\vec{B}$, not between the plane and $\vec{B}$

POTENTIAL ENERGY

$U = -\vec{m}\cdot\vec{B} = -mB\cos\theta$

$\theta = 0$ stable ($-mB$), $\theta = \pi$ unstable ($+mB$); work to turn it $W = mB(\cos\theta_1 - \cos\theta_2)$

ROTATING CHARGE

$m = \frac{q\omega R^2}{2} = \frac{q}{2M}L$

charge q of mass M circling at angular speed ω gives $I = \frac{q\omega}{2\pi}$; for an electron orbit $\mu = \frac{e}{2m_e}L$

Moving-coil galvanometer. A coil of N turns, area A, hangs in the radial field of curved pole pieces. The field is always in the plane of the coil, so the torque is the maximum $NIAB$ at every deflection; a spring gives a restoring torque $k\phi$. At rest $NIAB = k\phi$, so the deflection is proportional to the current and the scale is uniform.

CURRENT SENSITIVITY

$\frac{\phi}{I} = \frac{NAB}{k}$

more turns, stronger field, weaker spring

VOLTAGE SENSITIVITY

$\frac{\phi}{V} = \frac{NAB}{kR}$

R = coil resistance; doubling N doubles R too, so voltage sensitivity does not rise

METERS

ammeter: shunt $S = \frac{I_g G}{I - I_g}$

voltmeter: $R = \frac{V}{I_g} - G$

shunt small and in parallel, series resistance large; worked out in the Current Electricity sheet

TRAP

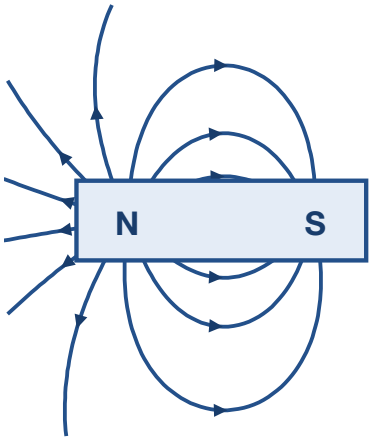
Torque on a coil: the angle is between the area vector (normal) and $\vec{B}$. When the field lies in the plane of the coil the normal is at 90°, so the torque is the maximum. Convert areas to m^2 first ($\text{cm}^2 \times 10^{-4}$).

CONNECTS TO

A current loop is the magnetic twin of the electric dipole: $\tau = mB\sin\theta$ and $U = -mB\cos\theta$ have the same form as $\tau = pE\sin\theta$ and $U = -pE\cos\theta$ in **Electrostatics**.

5 Bar magnet and the magnetic dipole

MAGNETIC DIPOLE A bar magnet behaves like a current loop or a short solenoid: it has a magnetic moment $\vec{M}$ pointing from its S pole to its N pole (inside the magnet). Isolated poles do not exist; the field lines are closed loops, so the net flux through any closed surface is zero, $\oint \vec{B}\cdot d\vec{A} = 0$.



lines leave N and enter S; inside the magnet they run from S to N

- Field lines leave at N, enter at S, and continue through the magnet from S to N.
- For a magnet of pole strength m_p and length $2l$, $M = m_p \cdot 2l$. Cut it **across**: each half has the same m_p but half the length. Cut it **along**: half the pole strength, same length. Either way each piece has $M/2$.
- As a solenoid: $M = NIA$ (turns times current times area).

ON THE AXIS (FAR FIELD)

$$B = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$$

along $\vec{M}$; $r \gg$ length of the magnet

ON THE EQUATORIAL LINE

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}$$

opposite to $\vec{M}$; half the axial value at the same r

IN A UNIFORM FIELD

$$\tau = MB\sin\theta, \quad U = -MB\cos\theta$$

small oscillations: $T = 2\pi\sqrt{\frac{I}{MB}}$ (I = moment of inertia)

KEY IDEA

The magnetic dipole field has the same shape as the electric dipole field: axial $2\times$ equatorial, both falling as $1/r^3$. Only the constants change, $\frac{1}{4\pi\varepsilon_0}p \to \frac{\mu_0}{4\pi}M$.

QUICK CHECK

Which is stronger at the same distance, the field on the axis or on the equator of a short bar magnet?

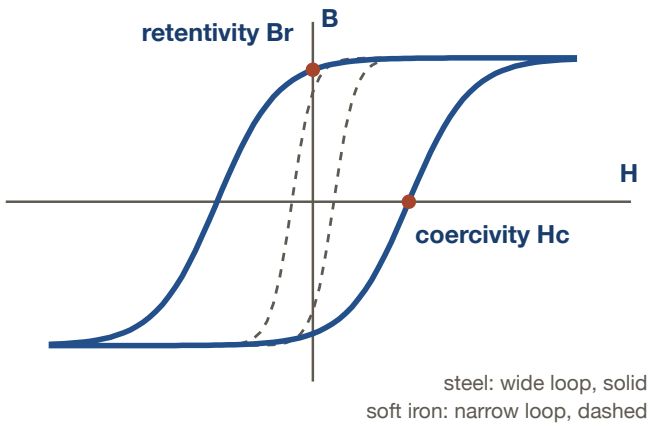
Answer: the axial field, by a factor of 2.

6 Magnetic materials

MAGNETISATION AND SUSCEPTIBILITY A material in a field $\vec{H}$ gets a magnetisation $\vec{M} = \chi\vec{H}$ and the total field is $\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0\mu_r\vec{H}$, with $\mu_r = 1 + \chi$. χ (susceptibility) is a pure number and sorts the materials into three classes.

DIA-, PARA- AND FERROMAGNETISM

PROPERTY	DIAMAGNETIC	PARAMAGNETIC	FERROMAGNETIC
Susceptibility χ	small, negative	small, positive	large and positive ($\gg 1$)
Relative permeability μ_r	slightly below 1	slightly above 1	much greater than 1
In a field	weakly repelled; moves to the weaker field	weakly attracted; moves to the stronger field	strongly attracted
Effect of temperature	almost none	falls with T: $\chi = C/T$ (Curie law)	becomes paramagnetic above the Curie temperature
Examples	bismuth, copper, gold, silver, lead, water	aluminium, sodium, calcium, oxygen, platinum	iron, cobalt, nickel, gadolinium, alnico



steel: wide loop, solid
soft iron: narrow loop, dashed

loop area = energy lost per cycle; steel keeps more B_r and needs a larger H_c

HYSTERESIS IN A FERROMAGNET

- Carry the field H up and down and B lags behind: after H returns to zero some B remains, the **retentivity** B_r; a reverse field, the **coercivity** H_c, is needed to bring B to zero.
- The area of the loop is the energy lost per unit volume per cycle, as heat.
- **Soft iron**: narrow loop, low retentivity and low coercivity, little loss; used for electromagnet and transformer cores. **Steel**, alnico: wide loop, high retentivity and coercivity; used for permanent magnets.

TRAP

Diamagnetic χ is negative, paramagnetic positive. A paramagnet's susceptibility halves when its absolute temperature doubles. Above the Curie temperature a ferromagnet is only paramagnetic. A superconductor is a perfect diamagnet, $\chi = -1$.

WHY IT WORKS

In a ferromagnet, groups of atoms (domains) have their moments lined up. A field makes domains aligned with it grow and the others rotate, so B is enormously larger than in vacuum; heating beyond the Curie point destroys the alignment.

7 Worked examples

SQUARE LOOP

A single-turn square loop of side 0.2 m carries 5 A. Field at its centre?

each side gives $\mu_0 I\sqrt{2}/2\pi a$; four sides: $\dfrac{2\sqrt{2}\,\mu_0 I}{\pi a} \approx 2.8 \times 10^{-5}$ T

PROTON IN A FIELD

A proton moves at 10^6 m/s perpendicular to 0.5 T. Radius and period?

$r = \dfrac{mv}{qB} \approx 2.1 \times 10^{-2}$ m; $T = \dfrac{2\pi m}{qB} \approx 1.3 \times 10^{-7}$ s

HELICAL PATH

An electron enters a 0.01 T field at 3×10^6 m/s, 60° to the field. Pitch?

$\dfrac{2\pi m v \cos 60^\circ}{qB} \approx 5.4 \times 10^{-3}$ m

ACCELERATED PROTON

A proton accelerated from rest through 1 kV enters a 0.1 T field perpendicularly. Radius?

$r = \dfrac{1}{B}\sqrt{\dfrac{2mV}{q}} \approx 4.6 \times 10^{-2}$ m

DEFLECTION ANGLE

A particle with orbit radius 2 cm enters a 1 cm wide field region perpendicular to its velocity. By what angle is it deflected?

$\sin\theta = \dfrac{d}{r} = \dfrac{1}{2}$, so $\theta = 30^\circ$

PARALLEL WIRES

Two long wires 10 cm apart carry 5 A and 3 A in opposite directions. Force per metre?

$2 \times 10^{-7} \times 15/0.1 = 3 \times 10^{-5}$ N/m, repulsive

TORQUE ON A COIL

A 100-turn coil of area $0.01\ \text{m}^2$ carries 2 A in a 0.5 T field; its normal makes 60° with the field. Torque?

$\tau = NIAB\sin 60^\circ = 100 \times 2 \times 0.01 \times 0.5 \times 0.866 \approx 0.87$ N m

GALVANOMETER

$N = 100$, $A = 1\ \text{cm}^2$, $B = 0.2$ T, $k = 10^{-6}$ N m/rad. Current sensitivity?

$NAB/k = 100 \times 10^{-4} \times 0.2/10^{-6} = 2000$ rad/A

BAR MAGNET FIELDS

$M = 2\ \text{A m}^2$. Fields on the axis and on the equator at 0.2 m?

$\dfrac{\mu_0}{4\pi}\dfrac{2M}{r^3} = 5 \times 10^{-5}$ T; $\dfrac{\mu_0}{4\pi}\dfrac{M}{r^3} = 2.5 \times 10^{-5}$ T

DIPOLE ENERGY

A dipole of $0.5\ \text{A m}^2$ lies at 30° to a 0.2 T field. Torque and potential energy?

$\tau = mB\sin 30^\circ = 0.05$ N m; $U = -mB\cos 30^\circ \approx -0.087$ J

CYCLOTRON

A proton cyclotron has $B = 1$ T and dee radius 0.5 m. Resonance frequency and final energy?

$f = \dfrac{qB}{2\pi m} \approx 15$ MHz; $K = \dfrac{q^2B^2R^2}{2m} \approx 1.9 \times 10^{-12}$ J ≈ 12 MeV

VELOCITY SELECTOR

Crossed fields $E = 2000$ V/m and $B = 0.5$ T. Which speed passes undeflected?

$v = \dfrac{E}{B} = 4 \times 10^3$ m/s, whatever the charge

8 Last-minute checklist

BEFORE YOU ANSWER

Which way does each piece's field point (thumb rule)? Is the angle measured from the normal or the plane? Is the charge positive or an electron? Are areas in m^2?

- Wire $\mu_0 I/2\pi d$; arc $\mu_0 I\theta/4\pi R$; loop centre $\mu_0 NI/2R$; solenoid $\mu_0 nI$; toroid $\mu_0 NI/2\pi r$ inside only.
- $F = q\vec{v} \times \vec{B}$ does no work; $r = mv/qB$, $T = 2\pi m/qB$; helix pitch uses $v\cos\theta$; velocity selector $v = E/B$.
- $F = BIl\sin\theta$; closed loop in a uniform field: zero force; parallel currents attract, $F/l = \mu_0 I_1 I_2/2\pi d$.
- $m = NIA$, $\tau = mB\sin\theta$ (θ from the normal), $U = -mB\cos\theta$; galvanometer $\phi/I = NAB/k$.
- Bar magnet: axial $2M \cdot 10^{-7}/r^3$, equatorial $M \cdot 10^{-7}/r^3$ (SI; opposite direction, half the size).
- Dia $\chi < 0$ small; para $\chi > 0$ small, $\chi \propto 1/T$; ferro $\chi \gg 1$, paramagnetic above the Curie temperature; soft iron for cores, steel for magnets.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**