JEE MAIN · PAPER 1 · PHYSICS

11 Electrostatics

Formula & concept sheet

Charges at rest: the force between them, the field and potential they make, how Gauss's law gives the field of symmetric bodies, how conductors behave, and how capacitors store energy. Each idea is explained first, then the formulas, standard results and traps.

INSIDE THIS SHEET

1	Charge and Coulomb's law	2
2	Electric field and field lines	3
3	Electric dipole	4
4	Flux and Gauss's law	5
5	Electric potential and potential energy	6
6	Conductors in electrostatics	7
7	Capacitors, dielectrics and energy	8
8	Worked examples	9
9	Last-minute checklist	10

READ ALONGSIDE NCERT XII · Electric Charges and Fields; Electrostatic Potential and Capacitance

HOW TO USE THIS SHEET

1. **Skim** — Read the Key ideas, then the formula cards.
2. **Memorise** — Learn the formulas and check every Trap box.
3. **Test** — Do the worked examples, then this chapter's PYQs.
4. **Revise** — Use the last-minute checklist before the exam.

Formula memorise · **Key idea** why it works · **Trap** marks lost here · **PYQ** how it is asked

Practise this chapter with real past-paper questions on Prep Vault. Written for revision, not as a first read.

CHAPTER 11

Electrostatics

NCERT XII · Electric Charges and Fields; Electrostatic Potential and Capacitance

Covers: charge and Coulomb's law · superposition · electric field and field lines · dipole · flux and Gauss's law · potential and potential energy · conductors · capacitors, dielectrics and energy.

1 Charge and Coulomb's law

COULOMB'S LAW The force between two stationary point charges acts along the line joining them. It is proportional to the product of the charges and inversely proportional to the square of their separation.

- Charge is **quantised** ($q = ne$, $e = 1.6 \times 10^{-19}$ C) and **conserved**: the total charge of an isolated system never changes. Sharing charge between conductors moves it around; it does not destroy it.
- Like charges repel, unlike charges attract. The two forces are equal and opposite (Newton's third law), whatever the charge sizes.
- Forces from several charges add as **vectors** (superposition); a third charge never changes the force between the first two.
- In a medium of dielectric constant K the force is divided by K.

COULOMB'S LAW

$$F = \frac{kq_1q_2}{r^2}$$

$k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2/C^2; $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m; repulsive if $q_1q_2 > 0$

IN A MEDIUM

$$F = \frac{kq_1q_2}{K\,r^2}$$

$K = 1$ for air or vacuum; water has a large K (about 80), which is why salt dissolves

SUPERPOSITION

$$\vec{F} = \vec{F}_{12} + \vec{F}_{13} + \dots$$

resolve into components first; on a line add with signs

THIRD CHARGE IN EQUILIBRIUM

like charges: $x = \dfrac{d\sqrt{q_1}}{\sqrt{q_1} + \sqrt{q_2}}$ from q_1, between them

unlike, $|q_1| < |q_2|$: $x = \dfrac{d\sqrt{|q_1|}}{\sqrt{|q_2|} - \sqrt{|q_1|}}$ from q_1, outside, away from q_2

from $\dfrac{kq_1}{x^2} = \dfrac{kq_2}{(d \mp x)^2}$; the sign of the third charge does not matter for its own equilibrium

TWO SUSPENDED CHARGED BALLS

$$\tan\theta = \frac{kq^2}{mg\,r^2}, \quad r = 2l\sin\theta$$

small θ: $r^3 = \dfrac{2kq^2l}{mg}$; each ball: $T\cos\theta = mg$, $T\sin\theta = F$

TRAP

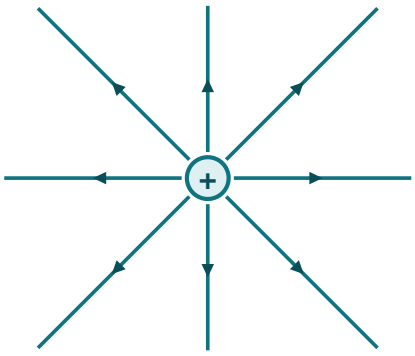
Force is a vector. For charges at the corners of a square or triangle, resolve and add components. Adding magnitudes, or using q with its sign inside $F = kq_1q_2/r^2$ to get a direction, are the usual errors.

SHORTCUT

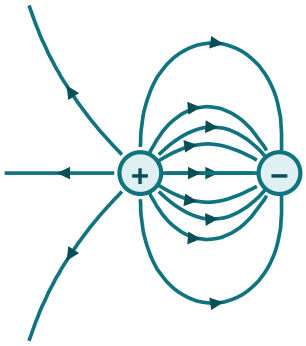
Balls hang at the same angle in air and in a liquid when $K = \dfrac{\rho_s}{\rho_s - \rho_l}$: the liquid cuts the force by K and the weight by buoyancy, $mg \to mg(1 - \rho_l/\rho_s)$.

2 Electric field and field lines

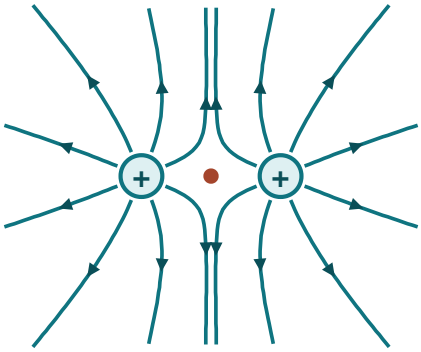
ELECTRIC FIELD The force per unit positive test charge at a point, $\vec{E} = \vec{F}/q_0$ (N/C or V/m). It exists whether or not a test charge is there.



Lines leave a $+$ charge radially and run out to infinity; arrows point along $\vec{E}$



Lines start on $+q$ and end on $-q$; none are closed loops



Like charges repel, so lines bend away from each other; the red dot is the neutral point where $E = 0$

- Field lines start on positive charge and end on negative charge (or run to infinity), never cross, and the tangent at a point is the direction of $\vec{E}$. Where lines are closer, the field is stronger.
- Lines leave a conductor's surface at right angles. In electrostatics they never form closed loops.
- For many charges the fields add as vectors. For a continuous charge, take a small element dq, write its $d\vec{E}$, let symmetry cancel the components that cancel, and integrate the rest.

POINT CHARGE

$$E = \frac{kq}{r^2}$$

radial: away from $+q$, towards $-q$

FORCE ON A CHARGE

$$\vec{F} = q\vec{E}$$

along $\vec{E}$ for $q > 0$, opposite for $q < 0$

CHARGE IN A UNIFORM FIELD

$$a = \frac{qE}{m}$$

motion like a projectile: across plates $y = \frac{qEx^2}{2mv^2}$

RING, ON ITS AXIS

$$E = \frac{kQx}{(R^2 + x^2)^{3/2}}$$

zero at the centre; maximum $\frac{2kQ}{3\sqrt{3}R^2}$ at $x = \frac{R}{\sqrt{2}}$

DISC, ON ITS AXIS

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

$R \to \infty$ gives the sheet, $\frac{\sigma}{2\varepsilon_0}$

SEMICIRCULAR ARC, AT THE CENTRE

$$E = \frac{2k\lambda}{R} = \frac{2kQ}{\pi R^2}$$

$\lambda = \frac{Q}{\pi R}$; components along the axis of symmetry add, the others cancel

FIELD OF A RING ON ITS AXIS (THE PATTERN FOR EVERY CONTINUOUS CHARGE)

$$dE = \frac{k\,dq}{R^2 + x^2}$$

each element of the ring is the same distance from the point

$$dE_x = dE\cos\alpha = \frac{k\,dq}{R^2 + x^2} \cdot \frac{x}{\sqrt{R^2 + x^2}}$$

components perpendicular to the axis cancel in pairs

$$E = \frac{kx}{(R^2 + x^2)^{3/2}} \int dq = \frac{kQx}{(R^2 + x^2)^{3/2}}$$

the integral of dq is the total charge

WHY IT WORKS

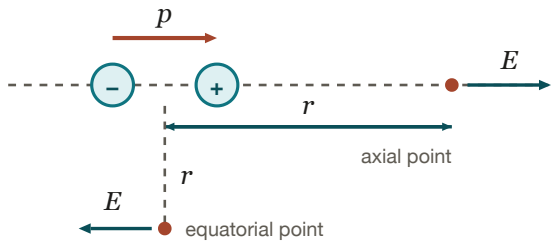
Lines cannot cross because $\vec{E}$ has one direction at each point. They are closer where E is larger because the number of lines through a small area is chosen to be proportional to E.

TRAP

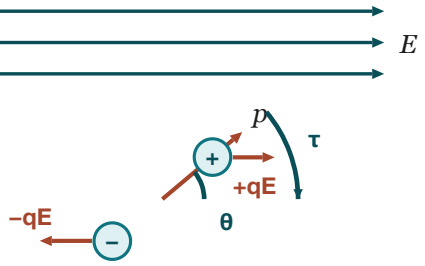
A field line is not the path of a charge. A released charge follows a line only if the line is straight, because it has inertia and accelerates along the tangent, not along the curve.

3 Electric dipole

ELECTRIC DIPOLE Two equal and opposite charges $\pm q$ a small distance d apart. Its dipole moment $\vec{p} = q\vec{d}$ points from $-q$ to $+q$ (unit C m). Its field falls off as $1/r^3$, faster than a single charge's $1/r^2$, because the two fields almost cancel.



On the axis $E = 2kp/r^3$ along $\vec{p}$; on the equator $E = kp/r^3$ opposite to $\vec{p}$



Equal and opposite forces $+qE$ and $-qE$ form a couple: $\tau = pE\sin\theta$ turns $\vec{p}$ towards $\vec{E}$

FIELD ON THE AXIS

$$E = \frac{2kp}{r^3}$$

along $\vec{p}$; far field, $r \gg d$

FIELD ON THE EQUATORIAL LINE

$$E = \frac{kp}{r^3}$$

opposite to $\vec{p}$; far field

POTENTIAL

$$V = \frac{kp\cos\theta}{r^2}$$

θ from the axis; $V = 0$ on the whole equatorial line

TORQUE IN A UNIFORM FIELD

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$\tau = pE\sin\theta$$

turns $\vec{p}$ towards $\vec{E}$; the net force is zero

POTENTIAL ENERGY

$$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$$

$\theta = 0$ stable ($-pE$); $\theta = \pi$ unstable ($+pE$)

SMALL OSCILLATIONS

$$T = 2\pi\sqrt{\frac{I}{pE}}$$

work to turn from θ_1 to θ_2: $W = pE(\cos\theta_1 - \cos\theta_2)$

TRAP

The $2kp/r^3$ and kp/r^3 results hold only for $r \gg d$. The equatorial field points **opposite** to $\vec{p}$. In a non-uniform field a dipole also feels a net force, $F = p\,dE/dx$.

QUICK CHECK

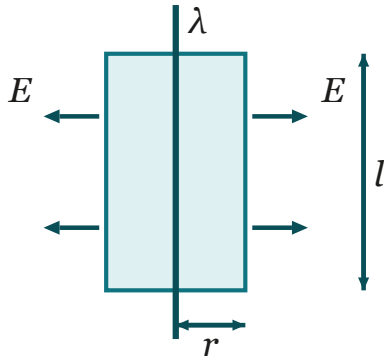
A dipole lies along a uniform field E. How much work turns it through 180°?

Answer: $W = pE(\cos 0 - \cos\pi) = 2pE$.

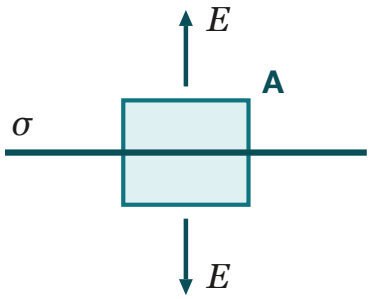
4 Flux and Gauss's law

GAUSS'S LAW The net electric flux through any closed surface equals the net charge enclosed divided by ε_0: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$. Flux through a small area is $\Phi = EA\cos\theta$ (θ between $\vec{E}$ and the normal).

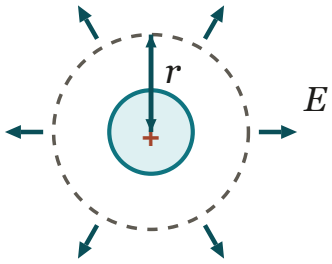
- Gauss's law is always true, but it **finds** E only when symmetry makes E constant (or zero) over the chosen surface.
- The flux counts only the **enclosed** charge, but $\vec{E}$ on the surface comes from **all** charges, inside and outside.
- Choose the Gaussian surface to match the symmetry: a coaxial cylinder for a line, a pillbox for a sheet, a concentric sphere for a sphere.



Cylinder of length l and radius r: E is radial and equal over the curved face, flux $= E\,2\pi rl$



Only the two end faces (area A each) carry flux: $2EA = \sigma A/\varepsilon_0$

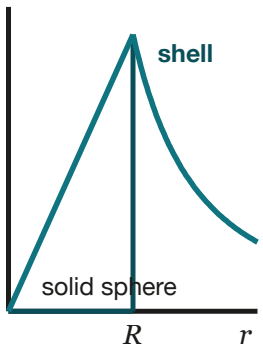


Sphere of radius r concentric with the charge: $E\,4\pi r^2 = q/\varepsilon_0$

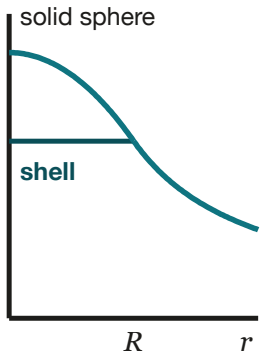
STANDARD RESULTS FROM GAUSS'S LAW

CHARGE DISTRIBUTION	FIELD	NOTES
Infinite line, λ per length	$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$	radial; falls as $1/r$
Infinite sheet, σ	$E = \frac{\sigma}{2\varepsilon_0}$	same on both sides, independent of distance
Two sheets, $+\sigma$ and $-\sigma$	$E = \frac{\sigma}{\varepsilon_0}$ between, 0 outside	a parallel-plate capacitor
Surface of a conductor	$E = \frac{\sigma}{\varepsilon_0}$ just outside, 0 inside	twice the sheet value (the field of the whole body adds)
Thin shell or conducting sphere, Q, R	$E = \frac{kQ}{r^2}$ for $r \geq R$; 0 inside	outside it acts like a point charge at the centre
Uniform solid sphere, Q, R	$E = \frac{kQ}{r^2}$ outside; $\frac{kQr}{R^3}$ inside	insulating sphere with charge spread through the volume

E against r



V against r



Both give E_R at R and fall as $1/r^2$ outside; inside, the shell has $E = 0$, V constant, and the solid sphere has $E \propto r$, $V = \frac{1}{2}(3 - r^2/R^2)V_R$

INSIDE A UNIFORM SOLID SPHERE

$$q_{enc} = Q\frac{r^3}{R^3}$$

charge in the sphere of radius r

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

Gauss, E constant on the sphere

$$E = \frac{kQr}{R^3}$$

linear in r

CUBE, CHARGE AT THE CENTRE

$$\Phi_{face} = \frac{q}{6\varepsilon_0}$$

all six faces equal; total $\frac{q}{\varepsilon_0}$

CUBE, CHARGE AT A CORNER

through the cube: $\frac{q}{8\varepsilon_0}$

each far face: $\frac{q}{24\varepsilon_0}$

the three faces that touch the corner get zero flux

DISC, CHARGE q ON ITS AXIS AT HEIGHT h

$$\Phi = \frac{q}{2\varepsilon_0}\left(1 - \frac{h}{\sqrt{h^2 + R^2}}\right)$$

hemisphere: $\frac{q}{2\varepsilon_0}$ $(h \to 0)$

KEY IDEA

Charge outside a closed surface adds no net flux: every line that enters also leaves. That is how a charge at a cube's corner can be split by symmetry into 8 equal cubes.

TRAP

$\sigma/2\varepsilon_0$ is for a thin sheet (field on each side). σ/ε_0 is just outside a conductor. They differ because the conductor's inside field is zero while a sheet's is not.

5 Electric potential and potential energy

POTENTIAL The work done per unit charge by an external agent in bringing a positive test charge slowly from infinity to the point: $V = W/q_0$ (volt, J/C). It is a **scalar**.

- Potential **difference** is what matters physically: $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l}$. The zero of potential is a choice.
- $E = -dV/dx$: the field points from high to low potential, and is larger where the potential changes fastest.
- Work done by the field moving q from A to B is $q(V_A - V_B)$ and does not depend on the path. **Equipotential surfaces** are always perpendicular to field lines; no work is done along one.

POTENTIAL OF COMMON BODIES (ZERO AT INFINITY)

BODY	POTENTIAL
Point charge	$V = \frac{kq}{r}$
Ring, on its axis	$V = \frac{kQ}{\sqrt{R^2+x^2}}$
Thin shell or conducting sphere, Q, R	$V = \frac{kQ}{r}$ for $r \geq R$; $V = \frac{kQ}{R}$ everywhere inside
Uniform solid sphere, Q, R	$V = \frac{kQ}{r}$ outside; $V = \frac{kQ(3R^2-r^2)}{2R^3}$ inside; at the centre $\frac{3}{2}\frac{kQ}{R}$
Dipole	$V = \frac{kp\cos\theta}{r^2}$
Infinite line	$V = -\frac{\lambda}{2\pi\varepsilon_0}\ln r + \text{const}$ (no zero at infinity)

FIELD AND POTENTIAL

$$E = -\frac{dV}{dx}$$

$$\Delta V = -\int E\,dx$$

given $E(x)$ integrate; given $V(x)$ differentiate; check the sign at the end

ENERGY OF A PAIR

$$U = \frac{kq_1q_2}{r}$$

positive for like charges, negative for unlike (zero at infinity)

ENERGY OF A SYSTEM

$$U = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_3}{r_{13}}\right)$$

each pair counted once; n charges give $\frac{n(n-1)}{2}$ pairs

WORK AND SPEED

$W = q\,\Delta V, \quad \frac{1}{2}mv^2 = q\,\Delta V$

an electron accelerated through V volts gains eV joules

CLOSEST APPROACH

$\frac{1}{2}mv^2 = \frac{kQq}{r_{min}}$

an α particle fired at a nucleus: speed zero at r_{min}

SELF-ENERGY OF A SPHERE

solid: $\frac{3kQ^2}{5R}$ · shell or conductor: $\frac{kQ^2}{2R}$

work to assemble the charge from infinity; $\frac{1}{2}\varepsilon_0 E^2$ integrated over space

TRAP

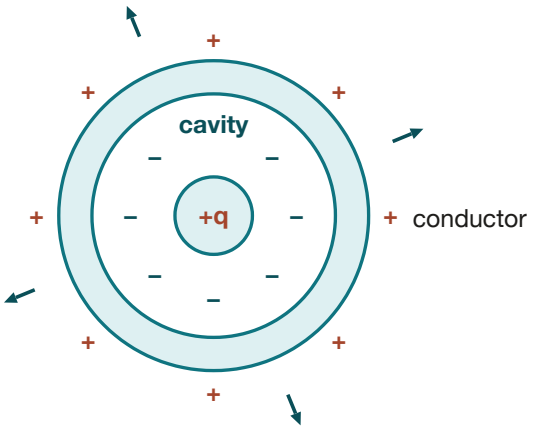
Potentials add as **scalars with sign**; fields add as vectors. $E = 0$ does not mean $V = 0$ (inside a charged shell $E = 0$ but $V = kQ/R$), and $V = 0$ does not mean $E = 0$ (the midpoint of $+q$ and $-q$).

QUICK CHECK

Two charges $+q$ sit at $x = \pm a$. Field and potential at the origin?

Answer: $E = 0$ (cancels), $V = \frac{2kq}{a}$ (adds).

6 Conductors in electrostatics



Inner $+q$ induces $-q$ on the inner surface and $+q$ on the outer; $E = 0$ in the metal, and outside the field is that of charge $+q$ at the centre

IN EQUILIBRIUM A CONDUCTOR HAS

- $\vec{E} = 0$ everywhere inside, and the whole body (and its surface) is at **one potential**
- any excess charge only on the **outer** surface; charge crowds at sharp points (large σ where the radius is small)
- field at the surface perpendicular to it, with $E = \sigma/\varepsilon_0$
- a hollow conductor **shields** its cavity from outside fields; a charge q placed in the cavity induces $-q$ on the cavity wall and $+q$ on the outer surface

WHY IT WORKS

Free charges move until the field they make cancels the applied field inside; any leftover field would keep pushing them. Hence $E = 0$ inside and V constant.

SPHERES JOINED BY A WIRE

$V_1 = V_2$

$Q \propto R, \quad \sigma \propto \frac{1}{R}$

the smaller sphere has the larger surface charge density and field

CONCENTRIC SHELLS

$V_{inner} = k\left(\frac{q}{a} - \frac{q}{b} + \frac{q+Q}{c}\right)$

inner sphere q, radius a; shell radii b, c, charge Q. Shell earthed: $V_{inner} = kq\left(\frac{1}{a} - \frac{1}{b}\right)$

PRESSURE ON A CHARGED SURFACE

$P = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0 E^2$

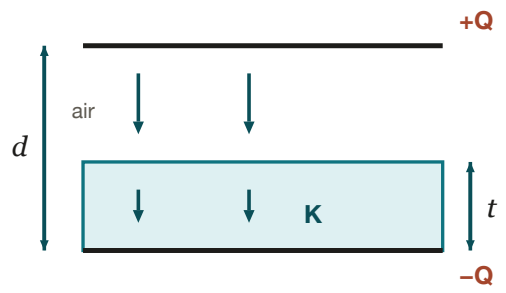
outward on a charged conductor (the average of the field just inside and outside)

TRAP

Earthing sets the **potential** to zero, not the charge. An earthed shell around a charged sphere carries only the induced $-q$ on its inner surface; its outer surface carries nothing.

7 Capacitors, dielectrics and energy

CAPACITANCE $C = Q/V$: charge stored per volt. It depends only on the **geometry** and the medium, never on Q or V. Unit farad (F).



Slab of thickness t and constant K in gap d: $C = \varepsilon_0 A/[(d-t)+t/K]$

PARALLEL PLATES

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

field between the plates

$$V = Ed = \frac{Qd}{\varepsilon_0 A}$$

uniform field

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

with a dielectric filling the gap: $\frac{K\varepsilon_0 A}{d}$

CAPACITANCE OF COMMON SHAPES

SHAPE	CAPACITANCE
Parallel plates, area A, gap d	$\frac{\varepsilon_0 A}{d}$; with a dielectric filling the gap $\frac{K\varepsilon_0 A}{d}$
Isolated sphere, radius R	$4\pi\varepsilon_0 R$
Concentric spheres, radii $a < b$	$\frac{4\pi\varepsilon_0\, ab}{b-a}$
Coaxial cylinders, length l	$\frac{2\pi\varepsilon_0\, l}{\ln(b/a)}$

SLAB OF THICKNESS t IN THE GAP d

$$C = \frac{\varepsilon_0 A}{d - t + t/K}$$

metal slab ($K \to \infty$): $\frac{\varepsilon_0 A}{d-t}$; the position of the slab does not matter

LAYERS IN SERIES (STACKED)

$$C = \frac{\varepsilon_0 A}{\sum t_i/K_i}$$

each layer is a capacitor in series with the same Q

SIDE BY SIDE (PARALLEL)

$$C = \frac{\varepsilon_0}{d}\sum K_i A_i$$

each part has the same V

COMBINATIONS

series: $\frac{1}{C} = \sum \frac{1}{C_i}$, same Q

parallel: $C = \sum C_i$, same V

opposite to resistors

ENERGY

$$U = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C} = \tfrac{1}{2}QV$$

density $u = \tfrac{1}{2}\varepsilon_0 E^2$ (times K with a dielectric)

FORCE BETWEEN PLATES

$$F = \frac{Q^2}{2\varepsilon_0 A} = \frac{\varepsilon_0 A V^2}{2d^2}$$

attractive; the same value at fixed Q or fixed V

What a dielectric does. Its molecules polarise in the field, leaving layers of induced (bound) charge on its faces that oppose the applied field. Inside, the field falls to E_0/K while the charge on the plates is unchanged, so for the same Q the potential difference falls and $C = Q/V$ rises by the factor K.

FIELD AND INDUCED CHARGE IN A DIELECTRIC

$$E = \frac{E_0}{K}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

σ = free charge density on the plate; a metal ($K \to \infty$) gives $E = 0$ and $\sigma_i = \sigma$

CHARGING FROM A BATTERY

work by battery $= QV$, stored $= \frac{1}{2}QV$

half of the battery's work is always lost as heat in the circuit, whatever the resistance

WHAT CHANGES? (THE TWO CASES TO SEPARATE EVERY TIME)

OPERATION	C	Q	V	E	U
Dielectric K inserted, battery on	$\times K$	$\times K$	same	same	$\times K$
Dielectric K inserted, battery off	$\times K$	same	$\div K$	$\div K$	$\div K$
Gap doubled, battery on	$\div 2$	$\div 2$	same	$\div 2$	$\div 2$
Gap doubled, battery off	$\div 2$	same	$\times 2$	same	$\times 2$

Battery connected: V fixed. Battery removed: Q fixed. The extra energy in the first row came from the battery; in the last row it is the work you did pulling the plates apart.

JOINING TWO CHARGED CAPACITORS

same polarity: $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

loss $= \dfrac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$

opposite polarity: $V = \dfrac{C_1V_1 - C_2V_2}{C_1 + C_2}$, loss $= \dfrac{C_1C_2(V_1 + V_2)^2}{2(C_1 + C_2)}$; the energy lost is heat and radiation

PLATES WITH UNEQUAL CHARGES

Q_1 and Q_2 on the two plates

facing surfaces carry $\pm\dfrac{Q_1 - Q_2}{2}$, outer surfaces $\dfrac{Q_1 + Q_2}{2}$ each; $V = \dfrac{Q_1 - Q_2}{2C}$

CAPACITOR NETWORKS

1. Label the nodes. Wires join nodes: merge them into one.
2. Series and parallel groups first; redraw after each step.
3. If $\dfrac{C_1}{C_2} = \dfrac{C_3}{C_4}$ the bridge is balanced: the middle capacitor carries no charge.
4. For isolated plates use conservation of charge, $\sum Q =$ constant, plus equal potentials on joined plates.

TRAP

C does not depend on the charge or the potential. Series reduces the total capacitance; parallel adds. μF means 10^{-6} F: convert before computing energy.

8 Worked examples

FIELD TO POTENTIAL

The field is $E_x = 2x$ (SI) along $+x$. Find $V(2) - V(0)$.

$$V(2) - V(0) = -\int_0^2 2x\,dx = -4 \text{ V}$$

EQUILIBRIUM POSITION

Charges $4q$ and q are d apart. Where can a third charge rest in equilibrium?

between them, $x = \dfrac{d\sqrt{4q}}{\sqrt{4q} + \sqrt{q}} = \dfrac{2d}{3}$ from the $4q$ charge

DIPOLE OSCILLATION

Two particles of mass m carrying $\pm q$, joined by a light rod of length d, rest in a field E. Small-angle period?

$I = 2m\left(\frac{d}{2}\right)^2 = \frac{md^2}{2}$, $\tau \approx qdE\,\theta$: $T = 2\pi\sqrt{\frac{md}{2qE}}$

CUBE CORNER

A charge q sits at one corner of a cube. Flux through each far face, and through the cube?

the cube is one eighth of a closed surface around q: $\frac{q}{8\varepsilon_0}$ in all, $\frac{q}{24\varepsilon_0}$ per far face

UNEQUAL PLATE CHARGES

Plates of a $1\ \mu$F capacitor carry $+2\ \mu$C and $+4\ \mu$C. Potential difference?

$|V| = \frac{|Q_1 - Q_2|}{2C} = \frac{2\times10^{-6}}{2\times10^{-6}} = 1$ V

PLATES PULLED APART

An isolated $5\ \mu$F capacitor holds $5\ \mu$C. Plates are pulled apart until $C = 2\ \mu$F. Work done?

$W = \frac{Q^2}{2}\left(\frac{1}{C_2} - \frac{1}{C_1}\right) = 12.5\left(\frac{1}{2} - \frac{1}{5}\right) = 3.75\ \mu$J

PARTIAL SLAB

Gap 2 mm, a 1 mm slab with $K = 4$ inside. Ratio of C with and without the slab?

$\frac{C'}{C} = \frac{d}{d - t + t/K} = \frac{2}{1.25} = 1.6$

JOINING CAPACITORS

A $2\ \mu$F capacitor at 100 V is joined to an uncharged $3\ \mu$F one. Final voltage and energy lost?

$V = \frac{200}{5} = 40$ V; loss $= 10 - 4 = 6$ mJ ($\frac{1}{2}\cdot 2\mu\cdot 100^2$ to $\frac{1}{2}\cdot 5\mu\cdot 40^2$)

JOINED SPHERES

Spheres of radius 1 cm and 2 cm share $3\ \mu$C and are joined by a wire. Charges and potential?

$Q \propto R$: $1\ \mu$C and $2\ \mu$C; $V = \frac{9\times10^9\times10^{-6}}{0.01} = 9\times10^5$ V

BATTERY ON, DIELECTRIC IN

A $10\ \mu$F capacitor on 100 V has a $K = 5$ dielectric inserted with the battery connected. New energy?

V fixed, $C \to 50\ \mu$F: $U = \frac{1}{2}(50\mu)(100)^2 = 0.25$ J (was 0.05 J)

9 Last-minute checklist

BEFORE YOU ANSWER

Vector or scalar? Which charges are enclosed, and which produce the field? Is the battery connected or removed? Are the units in SI (μF, cm)?

- Field and force are vectors; potential and energy are scalars with sign. $E = -dV/dx$.
- Gauss: point charge kq/r^2; line $2k\lambda/r$; sheet $\sigma/2\varepsilon_0$; conductor surface σ/ε_0; shell: 0 inside, $V = kQ/R$ inside.
- Dipole: $E_{axis} = 2kp/r^3$, $E_{eq} = kp/r^3$ (opposite $\vec{p}$); $\tau = pE\sin\theta$; $U = -pE\cos\theta$.
- $C = \varepsilon_0 A/d$; slab: $d \to d - t + t/K$; series adds $1/C$, parallel adds C; $U = \frac{1}{2}CV^2$.
- Battery on: V fixed (Q, U rise by K). Battery off: Q fixed (V, U fall by K).
- Joined conductors share potential ($Q \propto R$); joined capacitors share charge and lose energy.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**