JEE MAIN · PAPER 1 · PHYSICS

10 Oscillations & Waves

Formula & concept sheet

From a mass on a spring to a sound wave in an organ pipe: simple harmonic motion, its energy, springs and pendulums, then waves, standing waves, beats and the Doppler effect. Every idea is explained first, then the formulas, shortcuts and traps.

INSIDE THIS SHEET

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READ ALONGSIDE NCERT XI · Oscillations; Waves

HOW TO USE THIS SHEET

1. **Skim** — Read the Key ideas, then the formula cards.
2. **Memorise** — Learn the formulas and check every Trap box.
3. **Test** — Do the worked examples, then this chapter's PYQs.
4. **Revise** — Use the last-minute checklist before the exam.

Formula memorise · **Key idea** why it works · **Trap** marks lost here · **PYQ** how it is asked

Practise this chapter with real past-paper questions on Prep Vault. Written for revision, not as a first read.

CHAPTER 10

Oscillations & Waves

NCERT XI · Oscillations; Waves

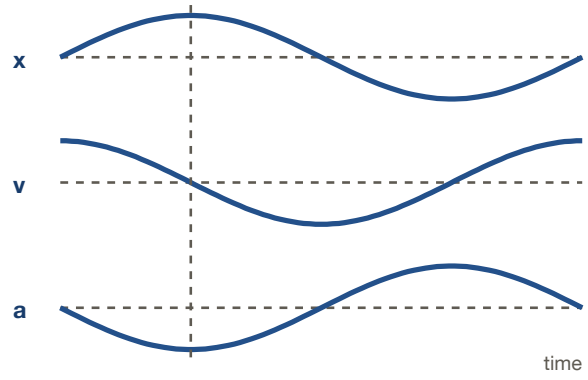
Covers: periodic motion · SHM and its equation · phase · spring and pendulum · energy in SHM · damped and forced oscillations · wave motion · progressive wave · superposition · reflection · standing waves in strings and organ pipes · beats · Doppler effect.

1 Periodic motion and SHM

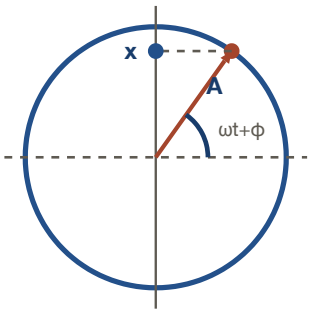
SIMPLE HARMONIC MOTION A motion in which the acceleration is proportional to the displacement from a fixed centre and always points towards it: $a = -\omega^2 x$.

A motion that repeats after a fixed time is **periodic**. SHM is the most important kind, because any small vibration about a stable equilibrium is SHM. Its law $F = -kx$ gives $\ddot{x} = -\omega^2 x$ with $\omega^2 = k/m$, and the solution is a sine or cosine.

- **Amplitude** A is the largest displacement. **Period** $T = 2\pi/\omega$ is the time for one full cycle; **frequency** $f = 1/T$; **angular frequency** $\omega = 2\pi f$ (rad/s).
- **Phase** $\omega t + \phi$ says where the particle is in its cycle. The **phase constant** ϕ is fixed by the starting position and the starting direction of motion.
- Speed is greatest at the mean position and zero at the ends. Acceleration is the reverse: zero at the mean position and largest ($\omega^2 A$) at the ends.



v is ahead of x by a quarter cycle; a is always opposite to x



The shadow of a point moving uniformly on the circle, on a diameter, does SHM.

speed on circle = Aω
one turn = one period

Use it for the time between two positions

WHY IT WORKS

$a = -\omega^2 x$ is exactly what the x-component of uniform circular motion obeys, because the centripetal acceleration $\omega^2 A$ points to the centre. That is why the circle gives the same x, v and a as SHM.

TRAP

The phase constant needs **both** $x(0)$ and the direction of $v(0)$. A particle at $x = A/2$ moving out and one moving in have phases of opposite sign. Sine arguments are in radians.

DISPLACEMENT

$x = A\sin(\omega t + \phi)$

or $A\cos(\omega t + \phi')$: the same motion

VELOCITY

$v = A\omega\cos(\omega t + \phi)$

$v = \omega\sqrt{A^2 - x^2}$

$v_{max} = A\omega$ at $x = 0$; $v = 0$ at $x = \pm A$

ACCELERATION

$a = -\omega^2 x$

$a_{max} = \omega^2 A$

towards the mean position; 0 at $x = 0$

TIME TO TRAVEL (USE THE REFERENCE CIRCLE)

PATH	TIME
$0 \to \frac{A}{2}$	$\frac{T}{12}$
$0 \to \frac{A}{\sqrt{2}}$	$\frac{T}{8}$
$\frac{A}{2} \to A$	$\frac{T}{6}$
$0 \to A$	$\frac{T}{4}$

from $x = A\sin\omega t$: $t = \frac{1}{\omega}\sin^{-1}\frac{x}{A}$

LINK BETWEEN v, a AND x

$$v^2 + \omega^2 x^2 = \omega^2 A^2$$

the v–x graph is an ellipse; the a–x graph is a straight line of slope $-\omega^2$

2 Is it SHM? Phase and combining SHMs

HOW TO PROVE A MOTION IS SHM

1. Displace the body a small distance x (or angle θ) from equilibrium.
2. Write the net restoring force F (or torque τ) in terms of that displacement.
3. If $F = -(\text{constant})\,x$, it is SHM and $\omega^2 = \dfrac{\text{constant}}{m}$. For rotation, $\tau = -C\theta$ and $\omega^2 = \dfrac{C}{I}$, so $T = 2\pi\sqrt{I/C}$.

PHASE CONSTANT FROM THE START

$$x(0) = A\sin\phi, \quad v(0) = A\omega\cos\phi$$

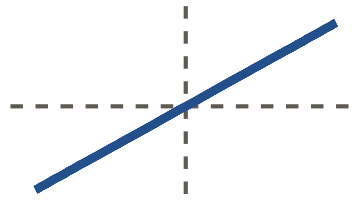
start at $+A$: $\phi = \frac{\pi}{2}$ · at $x = 0$ going $+$: $\phi = 0$ · at $x = 0$ going $-$: $\phi = \pi$ · at $-A$: $\phi = \frac{3\pi}{2}$

TWO SHMS, SAME DIRECTION AND ω

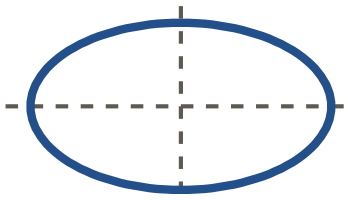
$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$$

δ = phase difference: $0 \to A_1 + A_2$ · $\pi \to |A_1 - A_2|$ · $\frac{\pi}{2} \to \sqrt{A_1^2 + A_2^2}$; the result is again SHM of the same ω

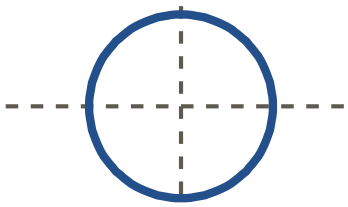
At right angles. Two SHMs of the same ω along x and y trace a straight line, an ellipse or a circle, depending on the phase difference δ (these are Lissajous figures). Different ω along x and y give more complex closed curves.



δ = 0: straight line



δ = π/2: ellipse



δ = π/2, A = B: circle

THESE ARE SHM

- $\sin\omega t \pm \cos\omega t = \sqrt{2}\sin(\omega t \pm \frac{\pi}{4})$
- $A_1\sin\omega t + A_2\cos\omega t$ (one ω)
- a pendulum or spring at small amplitude

PERIODIC BUT NOT SHM

- $\sin\omega t + \sin 2\omega t$, or $\cos\omega t + \cos 3\omega t$ (two ω's)
- $\sin^3\omega t$
- a ball bouncing elastically on the floor

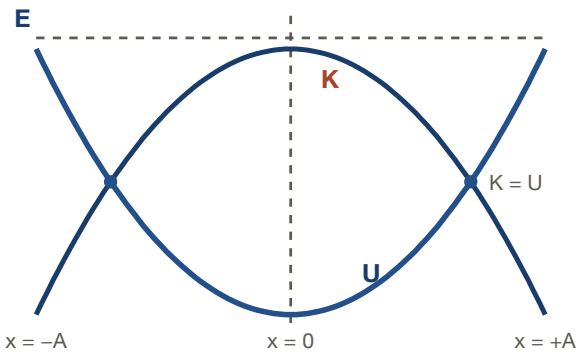
KEY IDEA

The test is $a \propto -x$, not "it repeats". $e^{-\omega^2 t^2}$ and $1 + \omega t + \omega^2 t^2$ are not even periodic. A sum of SHMs with one common ω is always SHM; with different ω it is not.

3 Energy in SHM

The total energy of a particle in SHM never changes: it only moves between kinetic energy (largest at the mean position) and potential energy (largest at the ends). It depends on the **square of the amplitude**, not on time.

- $E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2 = 2\pi^2mf^2A^2$. Double the amplitude and the energy becomes four times.
- Kinetic and potential energy each repeat **twice** per oscillation: their frequency is $2f$ and their period $T/2$. Their average over a cycle is $E/2$ each.



U and K are parabolas that add up to the flat line E

KINETIC ENERGY

$K = \frac{1}{2}m\omega^2(A^2 - x^2)$

largest, E, at $x = 0$

POTENTIAL ENERGY

$U = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}kx^2$

largest, E, at $x = \pm A$ (zero of U at $x = 0$)

RATIO AT A POSITION

$\frac{K}{U} = \frac{A^2 - x^2}{x^2}$

$K = U$ at $x = \pm\frac{A}{\sqrt{2}}$ · $K = 3U$ at $x = \pm\frac{A}{2}$

ENERGY AT A GLANCE

POSITION	KINETIC	POTENTIAL	SPEED	ACCELERATION
$x = 0$	E	0	$A\omega$	0
$x = \pm A/2$	$\frac{3}{4}E$	$\frac{1}{4}E$	$\frac{\sqrt{3}}{2}A\omega$	$\frac{1}{2}\omega^2 A$
$x = \pm A/\sqrt{2}$	$\frac{1}{2}E$	$\frac{1}{2}E$	$\frac{A\omega}{\sqrt{2}}$	$\frac{\omega^2 A}{\sqrt{2}}$
$x = \pm A$	0	E	0	$\omega^2 A$

TRAP

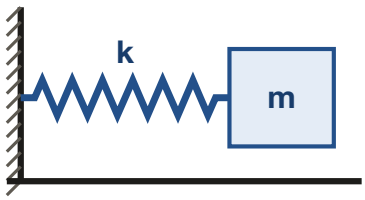
The frequency of K and U is **twice** that of x, v and a. "Average K over a cycle" is $E/2$, not E. Do not mix the two when asked for a frequency.

SHORTCUT

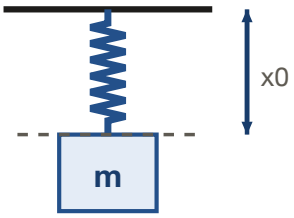
Work in x^2: $K = \eta E$ at $x = A\sqrt{1-\eta}$, and $U = \eta E$ at $x = A\sqrt{\eta}$. "Half the energy" is at $A/\sqrt{2}$ (about $0.71A$).

4 Spring–mass systems

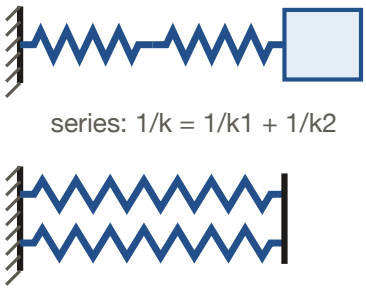
A block on a spring feels $F = -kx$, so $\omega = \sqrt{k/m}$. The period depends only on m and k: not on the amplitude, and (for a vertical spring) not on g, because gravity only shifts the equilibrium by mg/k.



F = –kx on the block



kx0 = mg at rest



series: 1/k = 1/k1 + 1/k2

parallel: k = k1 + k2

SPRING AND BLOCK

$T = 2\pi\sqrt{\frac{m}{k}}$

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

the same on a table, hung vertically or on a smooth incline

VERTICAL SPRING

$kx_0 = mg$

$T = 2\pi\sqrt{\frac{x_0}{g}}$

x_0 = extension at equilibrium; measure x from there

COMBINATIONS

parallel: $k = k_1 + k_2$

series: $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

two identical springs: $k_{series} = \frac{k}{2}$, $k_{par} = 2k$, so $T_{series} = 2T_{par}$. A spring cut into n equal parts: each part has $k' = nk$ (stiffness $\propto 1/\text{length}$)

TWO MASSES AND A SPRING

$$T = 2\pi\sqrt{\frac{\mu}{k}},\ \mu = \frac{m_1 m_2}{m_1 + m_2}$$

both free on a smooth surface

SPRING WITH ITS OWN MASS

$$T = 2\pi\sqrt{\frac{m + m_s/3}{k}}$$

ideal spring: $m_s = 0$

BLOCK ON A BLOCK

$$A_{max} = \frac{\mu_s g}{\omega^2}$$

largest amplitude before the upper block slips; $\omega^2 = \dfrac{k}{M + m}$

OTHER SYSTEMS THAT OSCILLATE (SMALL AMPLITUDE)

SYSTEM	TIME PERIOD	WHY
Liquid column of length L in a U-tube	$2\pi\sqrt{\dfrac{L}{2g}}$	weight of a $2x$ height difference restores
Cylinder floating with h submerged	$2\pi\sqrt{\dfrac{m}{A\rho g}} = 2\pi\sqrt{\dfrac{h}{g}}$	extra buoyancy $A\rho g\, x$ restores
Ball in a tunnel through the Earth	$2\pi\sqrt{R/g} \approx 84$ min	inside, $g \propto r$, so $F \propto -x$
Dipole in a field (small angle)	$2\pi\sqrt{\dfrac{I}{pE}}$	torque $-pE\theta$

TRAP

Series or parallel depends on what is **common**: the same extension (both ends tied together) is parallel; the same force (end to end) is series. Read the figure, not the wording.

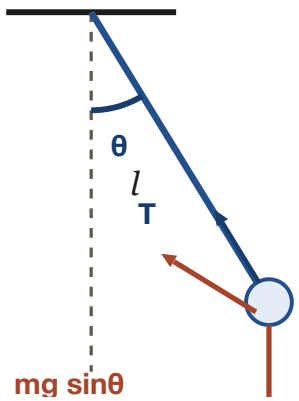
QUICK CHECK

A spring–block system has period T. The mass is quadrupled. New period?

Answer: $2T$, because $T \propto \sqrt{m}$.

5 Pendulums

For small angles (below about 10°) the restoring torque on a pendulum is proportional to its angle, so it swings in SHM. A simple pendulum's period does not depend on the bob's mass or on the amplitude; it depends on the length and on the **effective** gravity.



only the part of mg along the arc pulls the bob back: $F = -mg\sin\theta \approx -mg\theta$

PERIOD OF A SIMPLE PENDULUM

$$\tau = -mgl\sin\theta \approx -mgl\,\theta$$

small angle: $\sin\theta \approx \theta$

$$I\ddot{\theta} = -mgl\,\theta, \text{ with } I = ml^2$$

torque equation about the pivot

$$\omega^2 = \frac{g}{l}, \text{ so } T = 2\pi\sqrt{\frac{l}{g}}$$

the mass cancels

SIMPLE PENDULUM

$$T = 2\pi\sqrt{\frac{l}{g}}$$

seconds pendulum: $T = 2$ s, $l \approx 0.99$ m

PHYSICAL PENDULUM

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

$$l_{eq} = \frac{I}{md}$$

I about the pivot; d = pivot to centre of mass

LENGTH COMPARABLE TO R

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}$$

$l \to \infty$: $T = 2\pi\sqrt{R/g} \approx 84$ min

EFFECTIVE GRAVITY FOR A PENDULUM

SITUATION	g_{eff}	EFFECT ON T
Lift accelerating up (or slowing while going down) at a	$g + a$	shorter
Lift accelerating down at a	$g - a$	longer; free fall: no oscillation
Horizontal acceleration a (train, car)	$\sqrt{g^2 + a^2}$	equilibrium tilts, $\tan\theta = a/g$
Bob in a liquid (ρ_l liquid, ρ_b bob, no drag)	$g\left(1 - \frac{\rho_l}{\rho_b}\right)$	longer
Charged bob in a field E, charge q	$g \pm \frac{qE}{m}$	sign from the direction of the force

Find g_{eff} first, then $T' = T\sqrt{g/g_{eff}}$.

COMMON BODIES ABOUT A PIVOT

rod, end: $2\pi\sqrt{\frac{2L}{3g}}$

ring, rim: $2\pi\sqrt{\frac{2R}{g}}$

disc, rim: $2\pi\sqrt{\frac{3R}{2g}}$

from $I = \frac{1}{3}mL^2,\ 2mR^2,\ \frac{3}{2}mR^2$ and $d = \frac{L}{2},\ R,\ R$

SMALL CHANGES

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} - \frac{1}{2}\frac{\Delta g}{g}$$

heated by $\Delta\theta$: $\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$ (slower)

CLOCK GAINING OR LOSING

lost per day $= \frac{\Delta T}{T} \times 86400$ s

at height $h \ll R$: $\frac{\Delta T}{T} = \frac{h}{R}$ · at depth d: $\frac{d}{2R}$

WHY IT WORKS

The pull is $mg\sin\theta$ and the inertia is ml: both carry m, so it cancels. A heavier bob is pulled harder but is harder to move.

SHORTCUT

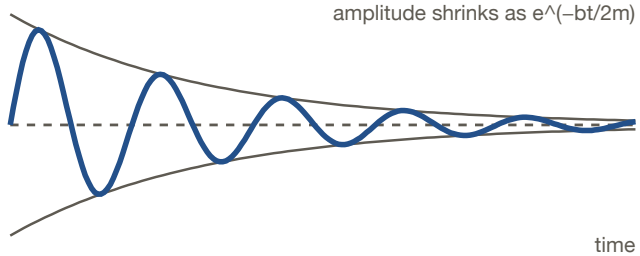
Liquid pendulum: $T' = T/\sqrt{1 - \rho_l/\rho_b}$. With $\rho_l = \rho_b/16$, $T' = \frac{4}{\sqrt{15}}T$. Torsion pendulum: $T = 2\pi\sqrt{I/C}$, with no g at all.

6 Damped and forced oscillations

SYLLABUS NOTE

Not in every recalled syllabus list, but it appears in older papers and coaching material. Learn the exponential decay and the resonance condition; skip the algebra of the forced amplitude.

Real oscillators lose energy to friction or drag. With a drag force $-bv$ the amplitude dies away exponentially while the oscillation continues at a slightly lower frequency. If a periodic driving force acts, the system settles to oscillate at the **driver's** frequency, and the amplitude is largest when that frequency matches the natural frequency: **resonance**.



the frequency barely changes while the amplitude dies away

DAMPED OSCILLATION

$$x = A_0 e^{-bt/2m}\sin(\omega' t + \phi)$$

$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$, a little below ω_0

AMPLITUDE AND ENERGY

$A = A_0 e^{-bt/2m} \cdot E = E_0 e^{-bt/m}$

$E \propto A^2$, so energy falls twice as fast

FORCED AND RESONANCE

resonance at $\omega_d \approx \omega_0$

sharper and taller for less damping

HOW IT IS ASKED

"Amplitude falls to half in n oscillations; how long to fall to $1/N$?" Count halvings: $t = \tau \log_2 N$ where τ is the time for one halving. No need to find b.

7 Wave motion and the progressive wave

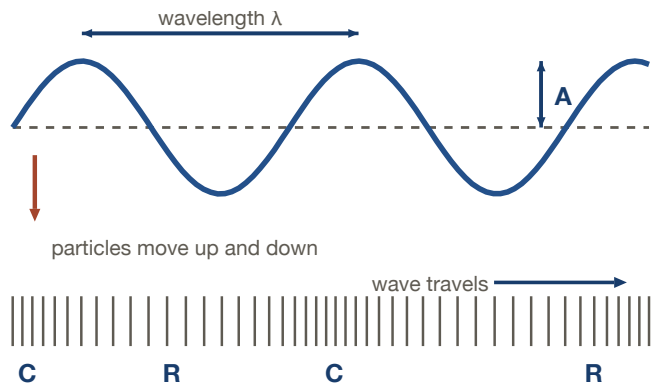
WAVE A disturbance that travels through a medium (or space) carrying energy and momentum without carrying the medium itself.

TRANSVERSE WAVE

- particles move across the direction of travel
- crests and troughs; string, light, ripples
- needs a medium that resists shear (solids) or surface tension

LONGITUDINAL WAVE

- particles move along the direction of travel
- compressions and rarefactions; sound in any medium
- needs a medium with volume elasticity (gas, liquid, solid)



top: transverse (particles move across the wave) · bottom: longitudinal, C = compression, R = rarefaction

- The **medium** sets the speed; the **source** sets the frequency. In a new medium f is unchanged and $\lambda = v/f$ changes.
- For $y = A\sin(kx - \omega t)$ the wave moves towards $+x$; for $y = A\sin(kx + \omega t)$, towards $-x$.
- Each particle does SHM with the wave's ω, lagging by kx at position x.
- Points a wavelength λ apart move together; points $\lambda/2$ apart are exactly opposite.

PROGRESSIVE WAVE

$$y = A\sin(kx - \omega t + \phi)$$

$$k = \frac{2\pi}{\lambda},\ \omega = 2\pi f$$

$v = \dfrac{\omega}{k} = f\lambda$; the minus sign gives travel in $+x$

PARTICLE MOTION

$$v_p = \frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$$

$$v_p = -v\frac{\partial y}{\partial x}$$

$v_{p,max} = A\omega$ (not v); $a_p = -\omega^2 y$

PHASE AND PATH DIFFERENCE

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \omega\Delta t$$

$\Delta x = \lambda \Leftrightarrow \Delta\phi = 2\pi$

SPEED OF WAVES IN DIFFERENT MEDIA

MEDIUM	WAVE SPEED	NOTES
Stretched string	$v = \sqrt{T/\mu}$	tension T, mass per unit length μ
Gas (Laplace)	$v = \sqrt{\dfrac{\gamma P}{\rho}} = \sqrt{\dfrac{\gamma RT}{M}}$	adiabatic, not isothermal; $v \propto \sqrt{T}$; at fixed T pressure does not matter
Liquid	$v = \sqrt{B/\rho}$	B = bulk modulus
Solid rod	$v = \sqrt{Y/\rho}$	Y = Young's modulus
Air near room temperature	$v \approx 331 + 0.61\,t$ m/s	t in °C; about 340 m/s at 15–20°C

POWER ON A STRING

$P = \frac{1}{2}\mu\omega^2 A^2 v$

energy per unit length $\frac{1}{2}\mu\omega^2 A^2$

INTENSITY OF SOUND

$I = 2\pi^2 f^2 A^2 \rho v$

$I \propto A^2 f^2$; point source: $I \propto \frac{1}{r^2}$, amplitude $\propto \frac{1}{r}$

SOUND LEVEL

$\beta = 10 \log_{10} \frac{I}{I_0}$ dB

$I_0 = 10^{-12}$ W/m^2; ten times the intensity adds 10 dB

HOW IT IS ASKED

Given $y = a\sin(bx - ct)$: read $A = a$, $k = b$, $\omega = c$, then $v = c/b$, $\lambda = 2\pi/b$, direction from the sign, and the largest particle speed ac. Or: change the tension or the medium and ask how v or λ changes.

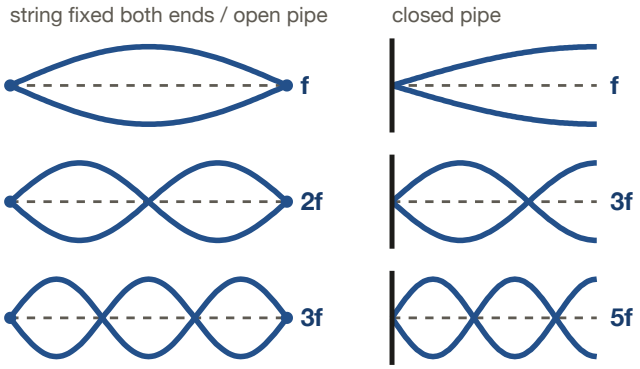
TRAP

Wave speed v and particle speed v_p are different things. Sound speed in a gas does not depend on pressure by itself: P/ρ is fixed once T is fixed. Intensity goes as amplitude **squared**.

8 Superposition, reflection and standing waves

When waves meet, their displacements simply add (**superposition**). A wave and its own reflection travel in opposite directions and make a **standing wave**: a fixed pattern of **nodes** (never move) and **antinodes** (swing most), with no net flow of energy.

- A **fixed end** reflects the wave upside down (phase change π), and is a node. A **free end** reflects without a phase change, and is an antinode.
- Node to node (or antinode to antinode) is $\lambda/2$; node to the next antinode is $\lambda/4$. Between two nodes all points move together; across a node the motion flips.



nodes (dots) are where the two envelopes touch · closed pipe: node at the wall, antinode at the open end

STRING FIXED AT BOTH ENDS

$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

$\lambda_n = \frac{2L}{n}$; the nth harmonic has n loops, $n+1$ nodes

FIXED AT ONE END ONLY

$f = \frac{(2n-1)v}{4L}$

odd harmonics only: f, $3f$, $5f$

LAWS OF A VIBRATING STRING

$f \propto \frac{1}{L}$, $f \propto \sqrt{T}$, $f \propto \frac{1}{\sqrt{\mu}}$

$\mu = \pi r^2 \rho$, so $f \propto \frac{1}{r\sqrt{\rho}}$

STANDING WAVE FROM TWO OPPOSITE WAVES

$y_1 = A\sin(kx - \omega t)$, $y_2 = A\sin(kx + \omega t)$	incident and reflected wave
$y = y_1 + y_2 = 2A\sin kx\cos\omega t$	$\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$
nodes: $x = \frac{n\lambda}{2}$ (where $\sin kx = 0$)	amplitude at x is $2A\lvert\sin kx\rvert$
antinodes: $x = \frac{(2n+1)\lambda}{4}$	largest swing, amplitude $2A$

SUPERPOSITION

$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$

constructive: $\Delta x = n\lambda$ · destructive: $\Delta x = (n + \frac{1}{2})\lambda$

INTENSITY RATIO

$\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2$

$I \propto A^2$; equal sources: $4I_0$ and 0

HARMONIC NAMES

1st harmonic = fundamental

2nd harmonic = 1st overtone

closed pipe: 3rd harmonic is the 1st overtone

METHOD

Given the standing-wave equation, read k from $\sin kx$, then $L = \frac{n\pi}{k}$ for a string fixed at both ends in its nth harmonic. Nodes are where $\sin kx = 0$.

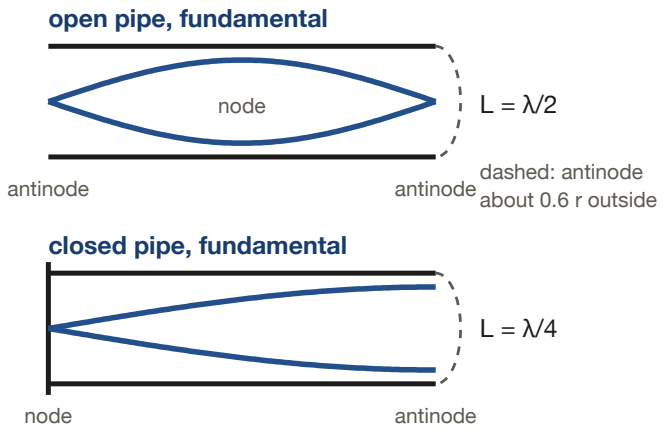
WHY IT WORKS

A fixed end cannot move, so the reflected wave must cancel the incident one there: an inverted pulse, phase change π. A free end returns it upright.

9 Sound and organ pipes

Sound is a longitudinal pressure wave. In a pipe the air column vibrates in standing waves. At a **closed** end the air cannot move: a displacement node (and a pressure antinode). At an **open** end the air moves freely: a displacement antinode (and a pressure node).

- An **open pipe** has antinodes at both ends, so it carries every harmonic $f, 2f, 3f, \ldots$. A **closed pipe** has a node at one end and an antinode at the other, so only **odd** harmonics $f, 3f, 5f, \ldots$, and its fundamental is half that of an open pipe of the same length.
- The antinode sits a little **outside** an open end, so the pipe behaves as slightly longer: the **end correction** $e \approx 0.6r$, with r the radius.



end correction $e \approx 0.6r$ lengthens each open end

OPEN PIPE

$f_n = \frac{nv}{2L}$ $(n = 1, 2, 3, \ldots)$

with end correction use $L + 2e$

CLOSED PIPE

$f_n = \frac{(2n-1)\,v}{4L}$

with end correction use $L + e$; odd harmonics only

SAME LENGTH

$f_{closed} = \frac{1}{2} f_{open}$

closed $1 : 3 : 5$; open $1 : 2 : 3$

RESONANCE TUBE (CLOSED, WATER LEVEL LOWERED)

$l_1 + e = \frac{\lambda}{4}$, $l_2 + e = \frac{3\lambda}{4}$

$\lambda = 2(l_2 - l_1) \cdot v = 2f(l_2 - l_1) \cdot e = \frac{l_2 - 3l_1}{2}$. The **difference** removes the end correction

WARM PIPE OR DIFFERENT GAS

$\frac{f'}{f} = \frac{v'}{v} = \sqrt{\frac{T'}{T}}$

kelvin; for another gas $v \propto \sqrt{\gamma/M}$

QUICK CHECK

An open pipe has fundamental 300 Hz. One end is closed. New fundamental, next harmonic?

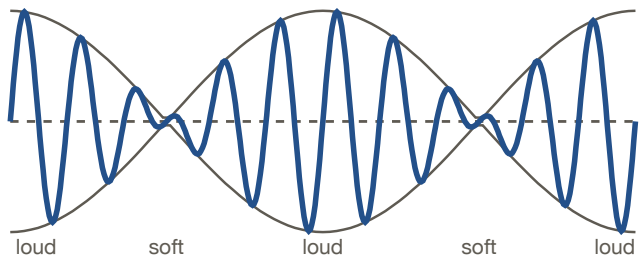
Answer: 150 Hz, then 450 Hz.

10 Beats and the Doppler effect

SYLLABUS NOTE

Doppler effect is not in every recalled syllabus list; it is included because it pairs with sound questions in older papers.

BEAT The rise and fall of loudness when two sounds of nearly equal frequency are heard together. The beat frequency is $|f_1 - f_2|$.



one beat = loud to loud: the loudness swells at $|f_1 - f_2|$ times a second

BEATS

$$f_{beat} = |f_1 - f_2|$$

beat period $= \dfrac{1}{f_{beat}}$; about 10 per second is the most a listener can count

RESULTANT OF TWO CLOSE TONES

$$y = 2A\cos(\pi \Delta f\, t)\sin(2\pi f_{av} t)$$

loudness follows the slow cosine envelope

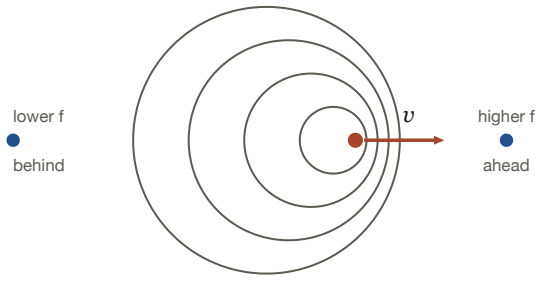
WHICH FORK IS HIGHER?

- ✓ wax on a fork **lowers** its frequency; filing it **raises** it
- ✓ if beats decrease after waxing the unknown fork, it was the higher one
- ✓ a known fork f_0 and n beats: the unknown is $f_0 \pm n$

TRAP

Beat frequency is the **difference**, never the sum. Counting loud-to-loud is one beat, not two.

Doppler effect. A moving source changes the wavelength in the medium, and a moving observer meets crests at a different rate. The speed of sound is set by the medium and does not change.



crests bunch up ahead of the moving source and spread out behind it

MOVING SOURCE, OBSERVER AT REST

crest spacing ahead: $\lambda' = \dfrac{v - v_s}{f}$

the source moves v_s/f between two crests

$$f' = \frac{v}{\lambda'} = f\,\frac{v}{v - v_s}$$

crests travel at v to the observer

DOPPLER (SOUND)

$$f' = f\,\frac{v - v_o}{v - v_s}$$

direction **source to observer** is positive; v_o, v_s signed along it

WITH WIND w

$$f' = f\,\frac{v + w - v_o}{v + w - v_s}$$

w along source to observer

REFLECTION FROM A MOVING WALL

$$f'' = f\,\frac{v + u}{v - u}$$

wall moving toward the source at u: it is first an observer, then a source

TRAP

Always check the sign: the pitch must **rise** when source and observer approach. The shift is not symmetric: approaching changes f more than receding at the same speed.

QUICK CHECK

A 1000 Hz source passes you at 20 m/s ($v = 340$ m/s). Pitch before and after?

Answer: 1062.5 Hz approaching, 944.4 Hz receding.

11 Worked examples

SPEED AT A DISPLACEMENT

SHM with amplitude 4 cm and period 2 s. Speed at $x = 2$ cm?

$\omega = \pi$ rad/s; $v = \pi\sqrt{4^2 - 2^2} = 2\sqrt{3}\,\pi \approx 10.9$ cm/s

CUT SPRING

A spring of $k = 200$ N/m is cut into 4 equal parts. A 0.5 kg block hangs on one. Find T.

$k' = 800$ N/m; $T = 2\pi\sqrt{0.5/800} = \dfrac{\pi}{20} \approx 0.157$ s

PENDULUM IN A LIFT

A lift accelerates upward at $g/3$. By what factor does a pendulum's period change?

$g_{eff} = \dfrac{4g}{3}$; $T' = T\sqrt{\dfrac{3}{4}} = \dfrac{\sqrt{3}}{2}\,T$

PENDULUM IN A LIQUID

A bob of density ρ swings in a liquid of density $\rho/16$. The period in air is T. Find it in the liquid (no drag).

$g_{eff} = g\left(1 - \dfrac{1}{16}\right) = \dfrac{15g}{16}$; $T' = \dfrac{4}{\sqrt{15}}T$

DAMPED OSCILLATOR

Frequency 5 Hz; amplitude halves every 10 oscillations. Time to fall to $\frac{1}{1000}$ of the original?

halving time $= 2$ s; $2^n = 1000 \Rightarrow n \approx 10$, so $t \approx 20$ s

STRING HARMONIC

$y = 0.3\sin(0.157x)\cos(200\pi t)$ (x in cm), string fixed at both ends, 4th harmonic. Length?

$k = 0.157$ cm^{-1}, $\lambda = 40$ cm; $L = 4 \cdot \dfrac{\lambda}{2} = 80$ cm

ORGAN PIPES

A closed pipe has fundamental 100 Hz ($v = 340$ m/s). Its length, and the fundamental of an open pipe of that length?

$L = \dfrac{v}{4f} = 0.85$ m; open: $\dfrac{v}{2L} = 200$ Hz

DOPPLER

A 1000 Hz source moves toward a stationary listener at 20 m/s ($v = 340$ m/s). Frequency heard?

$f' = \dfrac{1000 \times 340}{340 - 20} = 1062.5$ Hz

12 Last-minute checklist

SLIPS THAT COST MARKS

Length in **cm** inside $2\pi\sqrt{l/g}$. Angles in **degrees** inside a sine. Using ω where f is needed (a factor 2π). Two passes through the mean position make **one** period, not two.

BEFORE YOU ANSWER

Ask: is it SHM (is $a \propto -x$)? Which quantity oscillates at f, which at $2f$? Is g or k effective? Does the Doppler pitch rise when they approach?

- SHM test: $a \propto -x$ (or torque $\propto -\theta$). Read ω^2 from the constant.
- $E = \frac{1}{2}m\omega^2A^2$; K and U oscillate at $2f$; $K = U$ at $x = A/\sqrt{2}$.
- Spring: $T = 2\pi\sqrt{m/k}$; series adds $1/k$, parallel adds k; a shorter piece is stiffer.
- Pendulum: $T = 2\pi\sqrt{l/g_{eff}}$, independent of mass; physical pendulum $2\pi\sqrt{I/mgd}$.
- Wave: $v = f\lambda = \omega/k$; string $\sqrt{T/\mu}$; sound $\sqrt{\gamma P/\rho}$ with $v \propto \sqrt{T}$.
- String and open pipe $f_n = nv/2L$; closed pipe odd harmonics $(2n-1)v/4L$; beats $|f_1 - f_2|$.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**