JEE MAIN · PAPER 1 · PHYSICS

09 Kinetic Theory of Gases

Formula & concept sheet

Gases as swarms of molecules: the ideal gas equation, how pressure and temperature arise from molecular motion, degrees of freedom and mean free path. Each idea is explained in plain words, then the formulas, then how it is used and asked.

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READ ALONGSIDE NCERT XI · Kinetic Theory

HOW TO USE THIS SHEET

1 **Skim**
Read the Key ideas, then the formula cards.

2 **Memorise**
Learn the formulas and check every Trap box.

3 **Test**
Do the worked examples, then this chapter's PYQs.

4 **Revise**
Use the last-minute checklist before the exam.

Formula memorise **Key idea** why it works **Trap** marks lost here **PYQ** how it is asked

Practise this chapter with real past-paper questions on Prep Vault. Written for revision, not as a first read.

CHAPTER 09

Kinetic Theory of Gases

NCERT XI · Kinetic Theory

Covers: ideal gas equation, gas laws · kinetic theory, pressure and temperature · molecular speeds · degrees of freedom, equipartition · specific heats · mean free path.

1 Ideal gas and the gas laws

An **ideal gas** is a gas of point-like molecules that do not attract each other and collide elastically; real gases behave this way at low pressure and high temperature. Its pressure, volume and temperature are tied together by one equation of state.

EQUATION OF STATE

$$PV = nRT = Nk_BT$$

$R = 8.314\ \mathrm{J\,mol^{-1}K^{-1}}$, $k_B = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$, $R = N_Ak_B$

DENSITY FORM

$$P = \frac{\rho RT}{M} = nk_BT$$

M molar mass in kg/mol; $n = N/V$ is the number density

STANDARD VALUES

$N_A = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$

22.4 L per mole at STP

at $0^\circ\mathrm{C}$ and $1\ \mathrm{atm}$; temperature always in kelvin

GAS LAWS

LAW	HELD FIXED	RELATION
Boyle	T, n	$PV =$ constant
Charles	P, n	$V/T =$ constant
Gay-Lussac	V, n	$P/T =$ constant
Avogadro	P, T	$V \propto n$
Dalton (mixture)	T, V	$P = P_1 + P_2 + \cdots$
Graham (diffusion)	T, P	rate $\propto 1/\sqrt{M}$

MIXTURE OF GASES

$$n = n_1 + n_2$$

$$M_{mix} = \frac{n_1M_1 + n_2M_2}{n_1 + n_2}$$

each gas fills the whole volume; partial pressure $P_i = n_iRT/V$

HOW IT IS ASKED

A change of state ($P_1V_1/T_1 = P_2V_2/T_2$), a leaking or heated cylinder, mixing of gases, or the number of molecules in a given volume.

2 Kinetic theory: pressure and temperature

Gas pressure is the average force per area from molecules bouncing off the walls. Assumptions: many molecules in random motion, negligible molecular volume, no forces between collisions, elastic collisions with the walls. Temperature measures the **average translational kinetic energy** of a molecule, whatever its mass.

PRESSURE

$$P = \tfrac{1}{3}\frac{N}{V}m\,v_{rms}^2 = \tfrac{1}{3}\rho\,v_{rms}^2$$

also $P = \tfrac{2}{3}\times$ (translational energy per unit volume)

MEAN KINETIC ENERGY

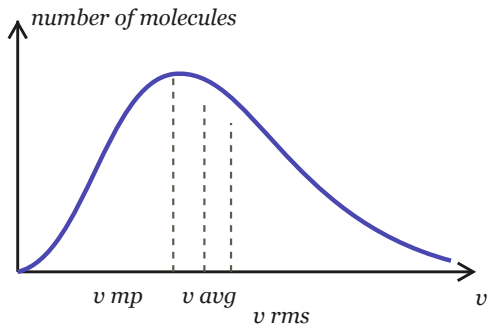
$$\tfrac{1}{2}mv_{rms}^2 = \tfrac{3}{2}k_BT$$

per molecule; $\tfrac{3}{2}RT$ per mole, the same for every gas

ROOT-MEAN-SQUARE SPEED

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_BT}{m}}$$

M in kg/mol; independent of P at fixed T



Peak v_{mp}, then v_{avg}, then v_{rms}: $\sqrt{2} : \sqrt{8/\pi} : \sqrt{3} \approx 1.41 : 1.60 : 1.73$.

THREE CHARACTERISTIC SPEEDS

$$v_{mp} = \sqrt{\frac{2RT}{M}}$$

$$v_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

all scale as $\sqrt{T/M}$

LINK TO THE SPEED OF SOUND

$$v_{sound} = \sqrt{\frac{\gamma RT}{M}}$$

$$\frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}}$$

TRAP

Convert Celsius to kelvin before using $\sqrt{T}$, and use molar mass in kg/mol. At fixed temperature the rms speed does not depend on pressure or volume; a given pressure change in a question is often a distractor.

3 Degrees of freedom and specific heats

A **degree of freedom** is an independent way a molecule can store energy (move along an axis, rotate about an axis, vibrate). The **law of equipartition** gives every quadratic energy term $\frac{1}{2}k_BT$ per molecule, so the internal energy of an ideal gas depends only on T.

EQUIPARTITION

$$U = \frac{f}{2}nRT$$

$\frac{1}{2}k_BT$ per degree of freedom

total energy, not only translational

HEAT CAPACITIES

$$C_V = \frac{f}{2}R, \quad C_P = \left(\frac{f}{2} + 1\right)R$$

$$\gamma = 1 + \frac{2}{f}$$

$C_P - C_V = R$

FROM MEASURED VALUES

$$f = \frac{2C_V}{R}$$

given C_P and C_V in J mol^{-1}K^{-1}, use $R = C_P - C_V$ if asked

GASES AT A GLANCE

GAS	DEGREES OF FREEDOM f	C_V	C_P	γ
Monatomic (He, Ar)	3 translational	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3}$
Diatomic, rigid (H_2, N_2)	3 trans. + 2 rot. $= 5$	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5}$
Diatomic with vibration (high T)	5 + 2 $= 7$	$\frac{7}{2}R$	$\frac{9}{2}R$	$\frac{9}{7}$
Non-linear polyatomic, rigid (CH_4)	3 trans. + 3 rot. $= 6$	$3R$	$4R$	$\frac{4}{3}$

a linear polyatomic such as CO_2 has 5 rigid degrees of freedom, like a diatomic gas

MIXTURE OF GASES

$$C_{V,mix} = \frac{n_1C_{V1} + n_2C_{V2}}{n_1 + n_2}$$

$$\gamma_{mix} = \frac{C_{V,mix} + R}{C_{V,mix}}$$

add moles, add nC_V, then $C_P = C_V + R$

MEAN FREE PATH

$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 n} = \frac{k_BT}{\sqrt{2}\,\pi d^2 P}$$

d molecular diameter, n number density; $\lambda \propto T/P$, independent of the molecular speed

HOW IT IS ASKED

Speed or temperature ratios from $v \propto \sqrt{T/M}$, the temperature at which v_{rms} equals the escape speed, f and γ of a gas or a mixture, and mean free path at a changed pressure or temperature.

4 Worked examples

SPEED AT A NEW TEMPERATURE

The rms speed of a gas is $200\ \mathrm{m\,s^{-1}}$ at $127^\circ\mathrm{C}$. At $227^\circ\mathrm{C}$ and double the pressure?

$v \propto \sqrt{T}$: $200\sqrt{\dfrac{500}{400}} \approx 224\ \mathrm{m\,s^{-1}}$; pressure does not matter

ENERGY PER MOLECULE

Average translational kinetic energy of a molecule at $300\ \mathrm{K}$, and per mole?

$\frac{3}{2}k_BT = 6.2 \times 10^{-21}\ \mathrm{J}$; per mole $\frac{3}{2}RT \approx 3740\ \mathrm{J}$

ESCAPE SPEED

Temperature at which v_{rms} of hydrogen ($M = 2 \times 10^{-3}\ \mathrm{kg\,mol^{-1}}$) equals $11.2\ \mathrm{km\,s^{-1}}$?

$$T = \frac{Mv^2}{3R} = \frac{2 \times 10^{-3} \times (11200)^2}{3 \times 8.31} \approx 1.0 \times 10^4\ \mathrm{K}$$

DEGREES OF FREEDOM

A gas has $C_V = 20.8\ \mathrm{J\,mol^{-1}K^{-1}}$. Degrees of freedom and γ?

$$f = \frac{2C_V}{R} = \frac{2 \times 20.8}{8.31} = 5;\ \gamma = 1 + \frac{2}{5} = \frac{7}{5}$$

GAS MIXTURE

Two moles of helium and one mole of hydrogen are mixed. γ of the mixture?

$$C_V = \frac{2 \cdot \frac{3}{2}R + \frac{5}{2}R}{3} = \frac{11}{6}R;\ C_P = \frac{17}{6}R;\ \gamma = \frac{17}{11} \approx 1.55$$

MEAN FREE PATH

Air at $300\ \mathrm{K}$, $1\ \mathrm{atm}$, molecular diameter $3 \times 10^{-10}\ \mathrm{m}$. Mean free path?

$$n = \frac{P}{k_BT} = 2.4 \times 10^{25}\ \mathrm{m^{-3}};\ \lambda = \frac{1}{\sqrt{2}\pi d^2 n} \approx 1.0 \times 10^{-7}\ \mathrm{m}$$

GRAHAM'S LAW

Rates of diffusion of hydrogen and oxygen under the same conditions?

$$\frac{r_{H_2}}{r_{O_2}} = \sqrt{\frac{32}{2}} = 4$$

GAS LAW

A gas at $300\ \mathrm{K}$ and $1\ \mathrm{atm}$ is heated to $450\ \mathrm{K}$ in a rigid container. Final pressure?

$$P_2 = P_1\frac{T_2}{T_1} = 1 \times \frac{450}{300} = 1.5\ \mathrm{atm}$$

KEY IDEA

Everything in this chapter comes from two facts: $PV = nRT$, and temperature is the average translational energy ($\frac{3}{2}k_BT$). Speeds follow, and equipartition then extends the energy to rotation and vibration.

5 Last-minute checklist

- Temperature in kelvin, molar mass in kg/mol, whenever a square root of T/M appears.
- $v_{rms} > v_{avg} > v_{mp}$, in the ratio $1.73 : 1.60 : 1.41$; none depends on pressure at fixed T.
- $U = \frac{f}{2}nRT$ counts every degree of freedom; $\gamma = 1 + 2/f$.
- Mixture: add the moles and the nC_V values, then find γ.
- Mean free path grows with temperature and falls with pressure.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**