06

JEE MAIN · PAPER 1 · PHYSICS

Gravitation

Formula & concept sheet

Newton's law of gravitation, how gravity varies, satellites, escape speed and Kepler's laws. Each idea is explained in plain words, then the formulas, then how it is used and asked.

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READ ALONGSIDE NCERT XI · Gravitation

HOW TO USE THIS SHEET

1 **Skim**
Read the Key ideas, then the formula cards.

2 **Memorise**
Learn the formulas and check every Trap box.

3 **Test**
Do the worked examples, then this chapter's PYQs.

4 **Revise**
Use the last-minute checklist before the exam.

Formula memorise · **Key idea** why it works · **Trap** marks lost here · **PYQ** how it is asked

Practise this chapter with real past-paper questions on Prep Vault. Written for revision, not as a first read.

CHAPTER 06

Gravitation

NCERT XI · Gravitation

Covers: law of gravitation · field and potential · variation of g · gravitational potential energy · satellites · escape speed · Kepler's laws.

1 Law of gravitation, field and potential

Every two masses attract along the line joining them, with a force proportional to the product of the masses and to $1/r^2$. The **field** is the force per unit mass; the **potential** is the work per unit mass to bring it from infinity, so it is negative.

NEWTON'S LAW

$$F = \frac{Gm_1m_2}{r^2}$$

$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$; always attractive

FIELD AND POTENTIAL (POINT MASS)

$$g = \frac{GM}{r^2}, \qquad V = -\frac{GM}{r}$$

$$\vec{g} = -\frac{dV}{dr}$$

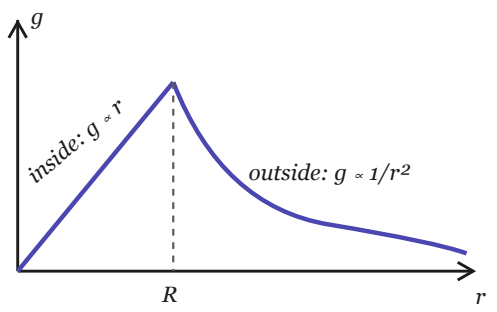
field is the slope of potential, pointing towards the mass

POTENTIAL ENERGY

$$U = -\frac{Gm_1m_2}{r}$$

$$W = \Delta U$$

for several masses add over every pair; zero at infinity



The field of a uniform solid sphere is largest at the surface.

FIELD AND POTENTIAL OF A SPHERE (MASS M, RADIUS R)

POSITION	FIELD	POTENTIAL
Point mass, or outside any sphere	$\frac{GM}{r^2}$	$-\frac{GM}{r}$
Thin shell, inside	0	$-\frac{GM}{R}$
Solid sphere, inside	$\frac{GMr}{R^3}$	$-\frac{GM(3R^2-r^2)}{2R^3}$
Solid sphere, centre	0	$-\frac{3GM}{2R}$

RING ON ITS AXIS

$$g = \frac{GMx}{(R^2+x^2)^{3/2}}$$

zero at the centre of the ring; largest at $x = R/\sqrt{2}$

CAVITY IN A UNIFORM SPHERE

$$g = \tfrac{4}{3}\pi G\rho\, d$$

uniform inside the cavity, directed along the line from the sphere's centre to the cavity's centre; d their separation

HOW IT IS ASKED

Field at a point from a sphere with a shell around it (add the contributions that apply at that distance), force or energy between masses, potential energy of a system of particles, and field in a cavity.

2 Acceleration due to gravity

At the surface $g = GM/R^2$. It falls going up (like $1/r^2$) and going down (linearly, to zero at the centre), and it varies with latitude because the Earth spins.

AT THE SURFACE

$$g = \frac{GM}{R^2} = \tfrac{4}{3}\pi G\rho R$$

$\approx 9.8\ \mathrm{m\,s^{-2}}$

AT A HEIGHT

$$g_h = \frac{g}{(1+h/R)^2}$$

$$\approx g\left(1 - \frac{2h}{R}\right)$$

the approximation needs $h \ll R$

AT A DEPTH

$$g_d = g\left(1 - \frac{d}{R}\right)$$

uniform-density Earth; zero at the centre

WITH LATITUDE

$$g_\lambda = g - \omega^2 R \cos^2 \lambda$$

smallest at the equator, equal to g at the poles

WEIGHTLESS AT THE EQUATOR

$$\omega = \sqrt{g/R}$$

a day of about 1.4 hours

TUNNEL THROUGH THE EARTH

$$T = 2\pi\sqrt{\frac{R}{g}} \approx 84.6 \text{ min}$$

simple harmonic motion, since $g \propto r$ inside

USEFUL VALUES

g halves at $h = (\sqrt{2} - 1)R$ and falls to a quarter at $h = R$: for heights comparable to R use the exact form, not $g(1 - 2h/R)$.

3 Satellites and escape speed

A satellite is in free fall around a planet and keeps missing it. Gravity supplies the centripetal force, which fixes the speed for a given radius: higher orbits are **slower**. The total energy of a bound orbit is negative.

ORBITAL SPEED

$$v_o = \sqrt{\frac{GM}{r}}$$

$\sqrt{gR} \approx 7.9 \text{ km s}^{-1}$ near the surface; no mass dependence

TIME PERIOD

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

near the surface $2\pi\sqrt{R/g} \approx 85$ min; $T^2 \propto r^3$

ENERGIES IN ORBIT

$$K = \frac{GMm}{2r},\ U = -\frac{GMm}{r}$$

$$E = -\frac{GMm}{2r}$$

$|E| = K$: the binding energy

ESCAPE SPEED

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

11.2 km s^{-1} for Earth; $v_e = \sqrt{2}\, v_o$

ESCAPE SPEED FROM DENSITY

$$v_e = R\sqrt{\frac{8\pi G\rho}{3}}$$

for planets of equal density, $v_e \propto R$

GEOSTATIONARY SATELLITE

$T = 24$ h, $r \approx 42\,400$ km

about $36\,000$ km up, over the equator

CHANGING THE ORBIT

$$\Delta E = \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

energy needed to move from radius r_1 to r_2

HIGHEST POINT OF A PROJECTILE

$$h = \frac{R}{\frac{2gR}{v^2} - 1}$$

launched upward at speed $v < v_e$ from the surface

BINARY STARS

$$T = 2\pi\sqrt{\frac{d^3}{G(M_1 + M_2)}}$$

separation d; each moves about the centre of mass

TRAP

Weightlessness in orbit is free fall, not zero gravity. With air drag a satellite **speeds up** as its orbit shrinks, though its total energy falls.

HOW IT IS ASKED

Ratios of speed, period or energy for two orbits, the minimum energy to escape or reach orbit, orbits around a given mass distribution, statements on weightlessness.

4 Kepler's laws

Kepler described planetary motion before Newton explained it. Law 1: planets move in ellipses with the Sun at one focus. Law 2: the line from the Sun to a planet sweeps equal areas in equal times. Law 3: the square of the period is proportional to the cube of the semi-major axis.

LAW OF AREAS

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$$

$$v_p r_p = v_a r_a$$

angular momentum is conserved; fastest at perihelion

LAW OF PERIODS

$$T^2 \propto a^3$$

$$T = 2\pi\sqrt{\frac{a^3}{GM}}$$

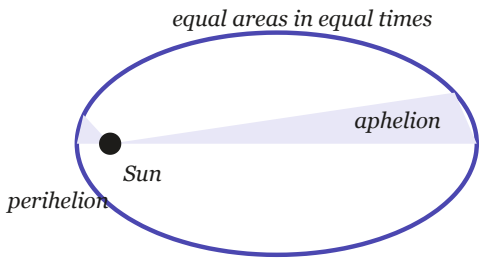
for a circle a is the radius

ENERGY AND SPEED ON AN ELLIPSE

$$E = -\frac{GMm}{2a}$$

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

a semi-major axis, r distance from the Sun



Fastest at perihelion, slowest at aphelion: $v_p r_p = v_a r_a$.

HOW IT IS USED

Compare two planets with $T_1^2/T_2^2 = a_1^3/a_2^3$: a planet four times farther takes eight times longer. Use $v_p r_p = v_a r_a$ to relate the speeds at the two ends of an orbit. Questions also give the area swept in a time, or ask the ratio of speeds at perihelion and aphelion.

5 Worked examples

G AT A HEIGHT

At what height above the Earth's surface does g fall to $g/9$?

$$\frac{g}{(1+h/R)^2} = \frac{g}{9} \Rightarrow 1 + \frac{h}{R} = 3, \text{ so } h = 2R$$

ESCAPE SPEED ON ANOTHER PLANET

A planet has $\frac{1}{8}$ of the Earth's mass and $\frac{1}{2}$ of its radius. Escape speed, if Earth's is $11.2\ \text{km s}^{-1}$?

$$v_e \propto \sqrt{M/R} = \sqrt{\tfrac{1}{8} \times 2} = \tfrac{1}{2}, \text{ so } 5.6\ \text{km s}^{-1}$$

ORBIT AT $4R$

A satellite orbits at radius $4R$ from the Earth's centre. Speed and period (Earth's surface orbit: $7.9\ \text{km s}^{-1}$, $84.6\ \text{min}$)?

$$v = \frac{7.9}{\sqrt{4}} \approx 3.95\ \text{km s}^{-1}; \quad T = 84.6 \times 4^{3/2} \approx 677\ \text{min}$$

BINDING ENERGY

Energy needed to free a $100\ \text{kg}$ satellite orbiting at $2R$ ($g = 9.8$, $R = 6.4 \times 10^6\ \text{m}$)?

$$\frac{GMm}{2r} = \frac{gRm}{4} \approx 1.57 \times 10^9\ \text{J}$$

DEPTH

At what depth does g fall to half its surface value?

$$g(1 - d/R) = g/2, \text{ so } d = R/2$$

KEPLER'S THIRD LAW

A planet orbits at four times the Earth's distance from the Sun. Its year?

$$T \propto a^{3/2}, \text{ so } T = 4^{3/2} = 8 \text{ Earth years}$$

ZERO OF THE FIELD

Masses M and $4M$ are a distance d apart. Where on the line between them is the net field zero?

$$\frac{GM}{x^2} = \frac{4GM}{(d-x)^2} \Rightarrow d - x = 2x, \text{ so } x = \frac{d}{3} \text{ from } M$$

ENERGY OF THREE MASSES

Three equal masses m sit at the corners of an equilateral triangle of side a. Potential energy of the system?

three pairs: $U = -\dfrac{3Gm^2}{a}$

6 Last-minute checklist

- Gravitational potential and potential energy are negative; the total energy of a bound orbit is $-K$.
- Inside a thin shell the field is zero but the potential is constant, not zero.
- Orbital speed does not depend on the satellite's mass; higher orbits are slower and have longer periods.
- Escape speed is independent of mass and direction; $v_e = \sqrt{2}\, v_o$ at the same radius.
- Use the exact g_h unless $h \ll R$.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**