JEE MAIN · PAPER 1 · PHYSICS

05 Rotational Motion

Formula & concept sheet

Centre of mass, moment of inertia, torque, angular momentum and rolling. Each idea is explained in plain words, then the formulas and tables, then how it is used and asked.

INSIDE THIS SHEET

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READ ALONGSIDE NCERT XI · System of Particles and Rotational Motion

HOW TO USE THIS SHEET

1. **Skim** — Read the Key ideas, then the formula cards.
2. **Memorise** — Learn the formulas and check every Trap box.
3. **Test** — Do the worked examples, then this chapter's PYQs.
4. **Revise** — Use the last-minute checklist before the exam.

Formula memorise · **Key idea** why it works · **Trap** marks lost here · **PYQ** how it is asked

Practise this chapter with real past-paper questions on Prep Vault. Written for revision, not as a first read.

CHAPTER 05

Rotational Motion

NCERT XI · System of Particles and Rotational Motion

Covers: centre of mass · moment of inertia and theorems · torque · angular momentum and its conservation · rolling · rigid-body equilibrium.

1 Centre of mass

The **centre of mass** is the mass-weighted average position. It moves as if all the mass sat there and all the external forces acted there; internal forces (an explosion, a collision) cannot change its motion.

POSITION

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

$$x_{cm} = \frac{1}{M}\int x\, dm$$

for a continuous body integrate over the mass

TWO PARTICLES

$$r_1 = \frac{m_2 d}{m_1 + m_2} \text{ from } m_1$$

the centre lies nearer the heavier mass

MOTION OF THE CENTRE

$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{M}$$

$$\vec{F}_{ext} = M\vec{a}_{cm}$$

no external force: v_{cm} stays constant

CENTRE OF MASS OF STANDARD SHAPES

BODY	DISTANCE
Semicircular ring	$2R/\pi$ from the centre
Semicircular disc	$4R/3\pi$ from the centre
Solid hemisphere	$3R/8$ from the centre
Hollow hemisphere	$R/2$ from the centre
Solid cone	$h/4$ from the base
Hollow cone, triangle lamina	$h/3$ from the base

HOW IT IS USED

A body with a cut-out: treat the hole as a body of **negative mass** and use the formula. An explosion in flight: the centre of mass keeps following the original path.

HOW IT IS ASKED

Centre of mass of a composite or cut body, the position of a boat or person after walking along it, and the shift of the centre in a collision.

2 Moment of inertia

The **moment of inertia** $I = \sum mr^2$ is the rotational counterpart of mass: it measures how hard it is to change a body's rotation, and it depends on the axis chosen. Mass far from the axis counts more.

CIRCULAR BODIES (MASS M, RADIUS R)

BODY AND AXIS	I
Ring, through centre, perpendicular to plane	MR^2
Ring, about a diameter	$\frac{1}{2}MR^2$
Disc or solid cylinder, its axis	$\frac{1}{2}MR^2$
Disc, about a diameter	$\frac{1}{4}MR^2$
Hollow cylinder, its axis	MR^2
Solid sphere, diameter	$\frac{2}{5}MR^2$
Hollow sphere, diameter	$\frac{2}{3}MR^2$

ROD, PLATE, AND VIA THE THEOREMS

BODY AND AXIS	I
Thin rod, perpendicular, through centre	$\frac{1}{12}ML^2$
Thin rod, perpendicular, through an end	$\frac{1}{3}ML^2$
Rectangular plate $a \times b$, perpendicular, centre	$\frac{1}{12}M(a^2+b^2)$
Ring or disc, tangent perpendicular to plane	$2MR^2$ ring, $\frac{3}{2}MR^2$ disc
Ring, tangent in its plane	$\frac{3}{2}MR^2$
Solid sphere, tangent	$\frac{7}{5}MR^2$

PARALLEL AXIS THEOREM

$$I = I_{cm} + Md^2$$

the axes must be parallel, and one must pass through the centre of mass

PERPENDICULAR AXIS THEOREM

$$I_z = I_x + I_y$$

only for flat (planar) bodies; x and y lie in the plane

RADIUS OF GYRATION

$$I = MK^2, \quad K = \sqrt{I/M}$$

the distance at which all the mass would give the same I

TRAP

The perpendicular-axis theorem fails for 3-D bodies, and the parallel-axis theorem must start from the axis through the **centre of mass**. For a body with a cut-out, subtract the removed part's moment of inertia about the same axis (using the parallel-axis theorem for it).

3 Torque and angular momentum

Torque is the turning effect of a force: force times its perpendicular distance from the axis. Net torque produces angular acceleration, just as net force produces linear acceleration. **Angular momentum** is conserved when no external torque acts: the skater who pulls her arms in spins faster.

TORQUE

$$\vec{\tau} = \vec{r} \times \vec{F}, \ \tau = rF\sin\theta$$

$$\tau = I\alpha$$

θ between $\vec{r}$ and $\vec{F}$; fixed axis

ANGULAR MOMENTUM

$$\vec{L} = \vec{r} \times \vec{p}, \ L = I\omega$$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

conserved if $\tau_{ext} = 0$

ROTATIONAL ENERGY AND POWER

$$K = \tfrac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

$$W = \tau\theta, \ P = \tau\omega$$

work done by a torque through angle θ

CONSERVATION OF ANGULAR MOMENTUM

$$I_1\omega_1 = I_2\omega_2$$

beads sliding on a rod, a person on a turntable; kinetic energy is **not** conserved, it changes as $L^2/2I$

LINEAR AND ROTATIONAL ANALOGUES

LINEAR	ROTATIONAL
$s,\ v,\ a$	$\theta,\ \omega,\ \alpha$
m	I
$F = ma$	$\tau = I\alpha$
$p = mv$	$L = I\omega$
$\tfrac{1}{2}mv^2$	$\tfrac{1}{2}I\omega^2$
$P = Fv$	$P = \tau\omega$

HOW IT IS ASKED

Moment of inertia of a composite or cut body, the new angular speed when beads move outwards or a person walks to the rim, torque from a given $K(\theta)$ or $\omega(t)$, and the angular acceleration for a given rotational energy.

PARTICLE IN CIRCULAR MOTION

$$L = mvr$$

about another point use the perpendicular distance d: $L = mvd$

ANGULAR IMPULSE

$$\int \tau\, dt = \Delta L$$

the rotational form of impulse equals change of momentum

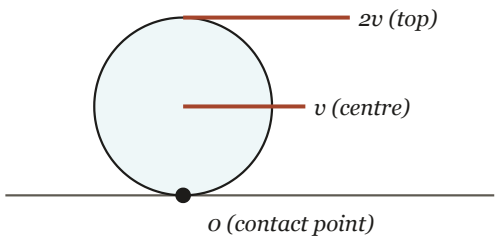
PARTICLE STRIKES A PIVOTED ROD

$$mvd = I\omega$$

angular momentum about the pivot is conserved; linear momentum is not (the pivot gives an impulse)

4 Rolling

In **pure rolling** the contact point does not slip, so $v = \omega R$. The kinetic energy is shared between moving the centre and spinning about it, which is why rolling bodies reach the bottom of a slope more slowly than sliding ones.



Pure rolling: $v = \omega R$; the contact point is instantaneously at rest.

ROLLING KINETIC ENERGY

$$K = \tfrac{1}{2}Mv^2\left(1 + \frac{K_g^2}{R^2}\right)$$

K_g is the radius of gyration; the rotational share is $\dfrac{K_g^2/R^2}{1 + K_g^2/R^2}$

ROLLING DOWN AN INCLINE

$$a = \frac{g\sin\theta}{1 + K_g^2/R^2}$$

$$\mu_{min} = \frac{\tan\theta}{1 + R^2/K_g^2}$$

static friction acts up the slope and does no work

ROLLING DOWN A SLOPE

BODY	K_g^2/R^2	A
Solid sphere	$\frac{2}{5}$	$\frac{5}{7}\,g\sin\theta$
Disc or solid cylinder	$\frac{1}{2}$	$\frac{2}{3}\,g\sin\theta$
Hollow sphere	$\frac{2}{3}$	$\frac{3}{5}\,g\sin\theta$
Ring or hollow cylinder	1	$\frac{1}{2}\,g\sin\theta$

order of arrival: solid sphere, disc, hollow sphere, ring, whatever the masses and radii

SPEED AND HEIGHT

$$v^2 = \frac{2gh}{1 + K_g^2/R^2}$$

at the bottom of a slope of height h

ROLLING UP A SLOPE

$$h = \frac{v^2}{2g}\left(1 + \frac{K_g^2}{R^2}\right)$$

height reached from speed v on the level

TRAP

Friction on a rolling body does **no work** in pure rolling (the contact point is at rest), so mechanical energy is conserved. If the body slips, friction is kinetic and energy is lost.

5 Fixed-axis rotation and equilibrium

For a rigid body rotating about a fixed axis, write $\tau = I\alpha$ about that axis. For a body in **equilibrium** both the net force and the net torque (about any point) are zero.

ROD RELEASED FROM HORIZONTAL

$$\alpha = \frac{3g}{2L}$$

$$\omega_{vertical} = \sqrt{\frac{3g}{L}}$$

rod of length L pivoted at one end, from rest

PULLEY WITH MASS

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2 + I/R^2}$$

string does not slip; $I/R^2 = M/2$ for a disc

EQUILIBRIUM

$$\sum \vec{F} = 0, \quad \sum \vec{\tau} = 0$$

take torques about the point that removes the most unknown forces

LADDER ON A SMOOTH WALL

$$\tan\theta \geq \frac{1}{2\mu}$$

uniform ladder at angle θ with the floor, friction μ at the floor only

TOPPLING

topples if the weight line leaves the base

$\tan\theta = \dfrac{b}{h}$ for a block on a tilting plane

b half base width, h height of the centre of mass

HOW IT IS USED

Choose the axis through the unknown force so its torque vanishes. For a rod on a hinge, use I about the hinge ($\frac{1}{3}ML^2$), not about the centre.

6 Worked examples

PARALLEL AXIS

A disc of mass $2\ \text{kg}$, radius $0.5\ \text{m}$. Moment of inertia about a tangent perpendicular to its plane?

$I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2} \times 2 \times 0.25 = 0.75\ \text{kg m}^2$

CONSERVATION OF ANGULAR MOMENTUM

A skater with $I = 4\ \text{kg m}^2$ spins at $2\ \text{rad s}^{-1}$ and pulls in her arms to $I = 1\ \text{kg m}^2$. New speed and energy change?

$\omega = \dfrac{4 \times 2}{1} = 8\ \text{rad s}^{-1}$; K: $8\ \text{J} \to 32\ \text{J}$

ROLLING SPHERE

A solid sphere rolls down a 30° incline ($g = 10$). Acceleration, and for a ring?

sphere $\frac{5}{7} \times 10 \times 0.5 \approx 3.57$; ring $\frac{1}{2} \times 10 \times 0.5 = 2.5\ \text{m s}^{-2}$

ROD RELEASED

A uniform rod of length $1.2\ \text{m}$ is pivoted at one end and released from horizontal ($g = 10$). α at release and ω at the vertical?

$\alpha = \dfrac{3 \times 10}{2 \times 1.2} = 12.5\ \text{rad s}^{-2}$; $\omega = \sqrt{\dfrac{30}{1.2}} = 5\ \text{rad s}^{-1}$

ROTATIONAL ENERGY

$I = 1.5\ \text{kg m}^2$ must reach $K = 1200\ \text{J}$. Angular speed?

$\omega = \sqrt{\dfrac{2K}{I}} = \sqrt{1600} = 40\ \text{rad s}^{-1}$

PULLEY WITH MASS

$3\ \text{kg}$ and $1\ \text{kg}$ over a disc pulley of mass $2\ \text{kg}$ ($g = 10$). Acceleration?

$a = \dfrac{(3-1)10}{3+1+1} = 4\ \text{m s}^{-2}$

ROLLING CYLINDER

A solid cylinder of mass $3\ \text{kg}$ rolls without slipping at $4\ \text{m s}^{-1}$. Total kinetic energy?

$K = \frac{1}{2}Mv^2\left(1 + \frac{1}{2}\right) = \frac{1}{2} \times 3 \times 16 \times 1.5 = 36\ \text{J}$

CLAY HITS A ROD

A $0.1\ \text{kg}$ clay ball at $10\ \text{m s}^{-1}$ sticks to the free end of a $1\ \text{kg}$, $1\ \text{m}$ rod pivoted at the other end. Angular speed just after?

$0.1 \times 10 \times 1 = \left(\frac{1}{3} + 0.1\right)\omega$, so $\omega \approx 2.31\ \text{rad s}^{-1}$

7 Last-minute checklist

- Name the axis before you write I, τ or L: every one of them depends on it.
- Parallel-axis theorem starts from the centre-of-mass axis; perpendicular-axis only for flat bodies.
- No external torque: L is conserved, kinetic energy is not.
- Rolling without slipping: $v = \omega R$ and static friction does no work.
- On a slope, the body with the smaller K_g^2/R^2 gets down first, whatever its mass or size.
- Equilibrium: forces and torques both sum to zero; take torques about the unknown force.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**