JEE MAIN · PAPER 1 · PHYSICS

04 Work, Energy & Power

Formula & concept sheet

How forces transfer energy: work, kinetic and potential energy, conservation, power and collisions. Each idea is explained in plain words, then the formulas, then how it is used and asked.

INSIDE THIS SHEET

1	Work and the work-energy theorem	2
2	Potential energy and conservation	2
3	Power	3
4	Collisions	4
5	What is conserved when	5
6	Worked examples	5
7	Last-minute checklist	5

READ ALONGSIDE NCERT XI · Work, Energy and Power

HOW TO USE THIS SHEET

1 **Skim**
Read the Key ideas, then the formula cards.

2 **Memorise**
Learn the formulas and check every Trap box.

3 **Test**
Do the worked examples, then this chapter's PYQs.

4 **Revise**
Use the last-minute checklist before the exam.

Formula – memorise **Key idea** – why it works **Trap** – marks lost here **PYQ** – how it is asked

CHAPTER 04

Work, Energy & Power

NCERT XI · Work, Energy and Power

Covers: work by constant and variable forces · work-energy theorem · potential energy, conservation · power · collisions in one and two dimensions.

1 Work and the work-energy theorem

A force does **work** only through the part of the displacement along it. Work is a scalar: positive when the force helps the motion, negative when it opposes it, zero when it is perpendicular. The **work-energy theorem** links work to motion: the net work on a body equals the change in its kinetic energy.

WORK BY A CONSTANT FORCE

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta$$

θ between force and displacement

WORK BY A VARIABLE FORCE

$$W = \int \vec{F} \cdot d\vec{r}$$

the area under the F–x graph, counted with sign

WORK-ENERGY THEOREM

$$W_{net} = \Delta K = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

true for all forces, conservative or not

KINETIC ENERGY

$$K = \tfrac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$p = \sqrt{2mK}$$

for equal momentum the lighter body has more energy

WORK DONE BY COMMON FORCES

FORCE	WORK
Gravity, lifting through h	$-mgh$ (gravity), $+mgh$ (you)
Spring, from x_1 to x_2	$\tfrac{1}{2}k(x_1^2 - x_2^2)$
Friction, sliding distance s	$-fs$ (heat produced)
Normal force, tension in a circle	0 (perpendicular to motion)
Pulling up $1/n$ of a chain off a table	$\dfrac{MgL}{2n^2}$

TRAP

Work by **static** friction can be non-zero (a crate on an accelerating truck), and the normal force on a moving wedge is not zero either. Use $W = \vec{F} \cdot \vec{s}$ with the displacement of the point where the force acts.

2 Potential energy and conservation

A force is **conservative** if its work depends only on the end points (gravity, spring, electric force). Then it has a potential energy U, and $K + U$ stays constant. Friction and drag are non-conservative: they change the total mechanical energy.

GRAVITATIONAL PE

$$U = mgh$$

near the Earth, measured from a chosen level

SPRING PE

$$U = \tfrac{1}{2}kx^2$$

x from the natural length

FORCE FROM PE

$$F = -\frac{dU}{dx}$$

the force points downhill on the U curve

CONSERVATION OF ENERGY

$$K_1 + U_1 = K_2 + U_2$$

$$W_{nc} = \Delta(K + U)$$

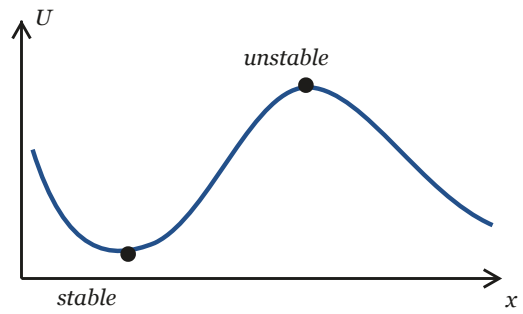
with friction: $W_{nc} = -fs$ is lost as heat

EQUILIBRIUM

$$\frac{dU}{dx} = 0$$

stable $\dfrac{d^2U}{dx^2} > 0$, unstable < 0

neutral equilibrium: U is constant



Slope of U is $-F$: zero at both marked points; a minimum is stable, a maximum unstable.

HOW IT IS USED AND ASKED

Energy conservation avoids solving the motion: use it when only gravity or springs act and you need a speed or a compression, not a time. Choose two positions, write $K + U$ at each and equate. Asked as the speed at the bottom of a slide or pendulum, the compression of a spring, or whether a bead reaches the top of a loop.

VERTICAL CIRCLE: STRING

top: $v \geq \sqrt{gr}$

bottom: $v \geq \sqrt{5gr}$

energy between top and bottom gives the $\sqrt{5}$

VERTICAL CIRCLE: LIGHT ROD

top: $v \geq 0$

bottom: $v \geq 2\sqrt{gr}$

a rod can push, so the top speed may be zero

PENDULUM BOB

$$v = \sqrt{2g\ell(1 - \cos\theta)}$$

θ from the vertical, released from rest at that angle

BLOCK ON A SMOOTH HEMISPHERE

leaves the surface where

$$\cos\theta = \frac{2}{3}$$

θ from the vertical, starting at the top

MASS DROPPED ON A SPRING

$$mg(h + x) = \tfrac{1}{2}kx^2$$

x is the maximum compression

MASS HUNG ON A SPRING

equilibrium $x = \dfrac{mg}{k}$

released from the natural length:

$$x_{max} = \frac{2mg}{k}$$

half the energy goes to the spring

LOOP-THE-LOOP

$$h_{min} = \tfrac{5}{2}\,r$$

least release height on a smooth track of radius r, so that $v_{bottom}^2 = 5gr$

ROUGH INCLINE, ENERGY FORM

$$v^2 = 2g(\sin\theta - \mu\cos\theta)\,s$$

from rest, after sliding a distance s down the slope

TRAP

With friction or an inelastic step, mechanical energy is **not** conserved (total energy is). Do not write $K + U =$ const across a collision.

3 Power

Power is the rate of doing work. At constant power a machine slows as the resisting force grows. Questions give $F(t)$ on a mass from rest ($v = \int a\,dt$, then $P = Fv$), a vehicle at a given power, or a pump.

POWER

$$P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$$

$$P_{av} = \frac{W}{t}$$

watt; $1\text{ hp} = 746\text{ W}$

EFFICIENCY

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

always below 100%

PUMPING AND LIFTING

$$P = \frac{mgh}{t} = \rho Q\,gh$$

Q is the volume flow rate

VEHICLE AT CONSTANT POWER

$$F = \frac{P}{v}, \quad v_{max} = \frac{P}{F_{resist}}$$

acceleration $a = \dfrac{P/v - F_{resist}}{m}$

POWER FROM THE MOTION

$$P = \frac{d}{dt}\left(\tfrac{1}{2}mv^2\right) = mva$$

when F is the only force doing work

4 Collisions

Momentum is conserved in every collision (there is no external impulse during the short contact). **Kinetic energy** is conserved only in an **elastic** collision. The **coefficient of restitution** e measures how elastic it is: $e = 1$ elastic, $e = 0$ perfectly inelastic (the bodies stick).

RESTITUTION

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

speed of separation over speed of approach, along the line of impact

PERFECTLY INELASTIC

$$v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

$$\Delta K = \frac{m_1 m_2}{2(m_1 + m_2)}(u_1 - u_2)^2$$

maximum loss of kinetic energy

EXPLOSION OR RECOIL

$$m_1 v_1 = m_2 v_2$$

$$K_1 : K_2 = m_2 : m_1$$

from rest: the lighter piece gets more energy; the released energy is $K_1 + K_2$

ELASTIC, ONE DIMENSION

$$v_1 = \frac{(m_1 - m_2)u_1 + 2m_2 u_2}{m_1 + m_2}$$

$$v_2 = \frac{(m_2 - m_1)u_2 + 2m_1 u_1}{m_1 + m_2}$$

both p and K conserved

GENERAL e, ONE DIMENSION

$$v_1 = \frac{(m_1 - em_2)u_1 + (1 + e)m_2 u_2}{m_1 + m_2}$$

$$v_2 = \frac{(m_2 - em_1)u_2 + (1 + e)m_1 u_1}{m_1 + m_2}$$

put $e = 1$ to recover the elastic result

SPECIAL ELASTIC CASES

SITUATION	RESULT
Equal masses, one at rest	velocities exchange
Heavy hits light at rest	light moves at about $2u$
Light hits heavy at rest	light rebounds with about u
KE passed to a target at rest	$\frac{4m_1 m_2}{(m_1 + m_2)^2}$
Equal masses, oblique, one at rest	final velocities at 90°

BALL AND THE FLOOR

rebound speed $v = eu$

height after n bounces $h\,e^{2n}$

dropped from h; total path $h\,\frac{1 + e^2}{1 - e^2}$

BALLISTIC PENDULUM

$$v_{bullet} = \frac{m + M}{m}\sqrt{2gh}$$

bullet embeds, then the block rises h

SPRING IN A COLLISION

$$\tfrac{1}{2}\mu(u_1 - u_2)^2 = \tfrac{1}{2}kx^2$$

at maximum compression both move together; $\mu = \frac{m_1 m_2}{m_1 + m_2}$

OBLIQUE HIT ON A WALL

normal: $v_n \to -e\,v_n$

tangential: unchanged

impulse $J = (1 + e)\,mu\cos\theta$

θ from the normal, smooth wall

JET ON A WALL

$F = \rho A v^2$ (absorbed)

$F = 2\rho A v^2$ (rebounds elastically)

force from the rate of change of momentum of the water

TRAP

In an oblique collision e acts only along the **normal**; the tangential component is unchanged. Momentum is conserved only along directions with no external impulse.

HOW IT IS ASKED

Mass ratio from the outgoing speeds in an elastic collision, speed after sticking together, e from bounce heights, KE lost, or the compression of a spring struck by a moving block.

5 What is conserved when

CONSERVATION AT A GLANCE

SITUATION	MOMENTUM	KINETIC ENERGY
Elastic collision	yes	yes
Inelastic collision	yes	no (some lost)
Perfectly inelastic (stick together)	yes	no (maximum loss)
Explosion	yes	no (increases)
Sliding with friction	no (external force)	no (heat)

WHICH TOOL?

Force and time: impulse and momentum. **Force and distance:** work and energy. **A collision:** momentum first. **Speed after a change of height or a spring:** energy. When two tools both fit, pick the one with fewer unknowns.

6 Worked examples

VARIABLE FORCE

A force $F = 3x^2 + 2$ (N) acts on a body from $x = 0$ to $x = 2$ m. Work done?

$$W = \int_0^2 (3x^2 + 2)\,dx = \left[x^3 + 2x\right]_0^2 = 12 \text{ J}$$

MASS DROPPED ON A SPRING

2 kg falls 0.5 m onto a spring $k = 800$ N m^{-1}, $g = 10$. Maximum compression?

$20(0.5 + x) = 400x^2 \Rightarrow 40x^2 - 2x - 1 = 0$, $x = \dfrac{2 + \sqrt{164}}{80} \approx 0.185$ m

ELASTIC COLLISION

2 kg at 6 m s^{-1} strikes a 4 kg body at rest, elastically. Velocities after?

$v_1 = \dfrac{(2-4)6}{6} = -2$, $v_2 = \dfrac{2 \cdot 2 \cdot 6}{6} = 4$ m s^{-1}; K: 36 J $\to 4 + 32 = 36$ J

BULLET AND BLOCK

A 10 g bullet at 400 m s^{-1} embeds in a 1.99 kg block at rest. Common speed and energy lost?

$v = \dfrac{0.01 \times 400}{2} = 2$ m s^{-1}; K: 800 J $\to 4$ J, lost 796 J

POWER OF AN ENGINE

A 20 kW engine pulls a vehicle at a steady 10 m s^{-1}. Force exerted?

$$F = \frac{P}{v} = \frac{20000}{10} = 2000 \text{ N}$$

VERTICAL CIRCLE

A bob on a 1 m string passes the top at the minimum speed. Its speed at the bottom ($g = 10$)?

$v_{top}^2 = gr = 10$; $v_{bot}^2 = v_{top}^2 + 4gr = 50$, $v \approx 7.07$ m s^{-1}

CHAIN ON A TABLE

A uniform cable of mass 4 kg and length 2 m has $\frac{1}{4}$ of its length hanging over the edge. Work to pull it fully up ($g = 10$)?

$$W = \frac{MgL}{2n^2} = \frac{4 \times 10 \times 2}{2 \times 16} = 2.5 \text{ J}$$

BOUNCING BALL

A ball dropped from 2 m has $e = 0.5$. Height after the first bounce and total path travelled?

$h_1 = e^2 h = 0.5$ m; path $= h\dfrac{1+e^2}{1-e^2} = 2 \times \dfrac{1.25}{0.75} \approx 3.33$ m

7 Last-minute checklist

- Work is $\vec{F} \cdot \vec{s}$: check the sign, and that the displacement is of the point where the force acts.
- A graph of F against x: work is the area, signed. A U curve: force is minus the slope.
- Collisions: write momentum first, then e or energy. Keep the sign of each velocity.
- For power problems, find v at the instant first, then $P = Fv$.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**