JEE MAIN · PAPER 1 · PHYSICS

02 Kinematics

Formula & concept sheet

Motion without the forces: vectors, straight-line motion, graphs, relative velocity, projectiles and circular motion. Each idea is explained in a few lines, then the formulas, then how it is used and asked.

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READ ALONGSIDE NCERT XI · Motion in a Straight Line; Motion in a Plane

HOW TO USE THIS SHEET

1. **Skim** — Read the Key ideas, then the formula cards.
2. **Memorise** — Learn the formulas and check every Trap box.
3. **Test** — Do the worked examples, then this chapter's PYQs.
4. **Revise** — Use the last-minute checklist before the exam.

Formula memorise · **Key idea** why it works · **Trap** marks lost here · **PYQ** how it is asked

Practise this chapter with real past-paper questions on Prep Vault. Written for revision, not as a first read.

CHAPTER 02

Kinematics

NCERT XI · Motion in a Straight Line; Motion in a Plane

Covers: vectors · motion in a straight line · graphs · relative velocity · projectile motion · circular motion.

1 Vectors

A **scalar** has only a size (distance, speed). A **vector** has size and direction (displacement, velocity, acceleration) and adds by the triangle rule, not by plain arithmetic.

RESULTANT OF TWO VECTORS

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

θ between them, tail to tail; α from $\vec{A}$. Range $|A - B|$ to $A + B$; for $\vec{A} - \vec{B}$ the cosine term is negative

RESOLUTION AND UNIT VECTOR

$$A_x = A\cos\theta,\ A_y = A\sin\theta$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|},\ |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

θ measured from the x-axis; components add independently

DOT AND CROSS PRODUCT

$$\vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

$$|\vec{A}\times\vec{B}| = AB\sin\theta$$

dot $= 0$ if perpendicular, cross $= 0$ if parallel (right-hand rule)

2 Motion in a straight line

In one dimension direction is just a sign: displacement and velocity carry it, distance and speed do not. Choose a positive direction, then use the equation that leaves out what you neither know nor need.

DEFINITIONS

$$v = \frac{dx}{dt},\quad a = \frac{dv}{dt} = v\frac{dv}{dx}$$

use $a = v\,dv/dx$ when a is given as a function of position

AVERAGE SPEED VS VELOCITY

$$\text{speed} = \frac{\text{total distance}}{\text{total time}},\quad \vec{v}_{av} = \frac{\Delta\vec{x}}{\Delta t}$$

speed $\geq |\vec{v}_{av}|$ always

EQUATIONS OF MOTION (UNIFORM ACCELERATION)

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

EXTRAS

$$s = \frac{u+v}{2}t$$

$$s_n = u + \frac{a}{2}(2n-1)$$

s_n = distance in the nth second

FREE FALL / THROW UP

$$H = \frac{u^2}{2g},\ T = \frac{2u}{g}$$

$$t = \sqrt{2h/g},\ v = \sqrt{2gh}$$

$g = 9.8$ or 10 as the paper says

KEY IDEA

Speeding up or slowing down depends on the **relative sign** of v and a, not on the sign of a. Same sign: speeding up. Opposite: slowing down.

TRAP

Equal **distances** at speeds v_1, v_2 give the harmonic mean $\dfrac{2v_1v_2}{v_1+v_2}$. Equal **times** give the arithmetic mean $\dfrac{v_1+v_2}{2}$.

3 Reading motion graphs

POSITION–TIME GRAPH

slope $= v$

straight line: v constant

horizontal line: at rest

a curve means $a \neq 0$; tangent slope is v

VELOCITY–TIME GRAPH

slope $= a$

area $=$ displacement

crosses the axis: turns back

for distance, count every area as positive

ACCELERATION–TIME GRAPH

area $= \Delta v$

$v_f = u +$ area

horizontal line: a constant

a jump in a is a kink, not a jump, in the v–t graph

DISTANCE VS DISPLACEMENT

$$d = \int |v|\,dt, \qquad \Delta x = \int v\,dt$$

they differ as soon as the particle turns back, so split the integral where $v = 0$

FROM REST, UNIFORM ACCELERATION

$$s_1 : s_2 : s_3 = 1 : 3 : 5$$

$$t_1 : t_2 : t_3 = 1 : (\sqrt{2}-1) : (\sqrt{3}-\sqrt{2})$$

first line: distances in equal times; second line: times for equal distances

4 Variable acceleration and chase problems

When a is not constant the three equations stop working. Integrate instead, using the rate as given: a function of time, of position or of speed.

ACCELERATION AS A FUNCTION OF TIME

$$v = u + \int a\,dt$$

$$x = x_0 + \int v\,dt$$

for $a = kt$: $v = u + \frac{1}{2}kt^2$, $x = ut + \frac{1}{6}kt^3$

ACCELERATION AS A FUNCTION OF POSITION

$$\int v\,dv = \int a\,dx$$

$$\tfrac{1}{2}(v^2 - u^2) = \int a\,dx$$

limits: the starting speed and position

ACCELERATION AS A FUNCTION OF SPEED

$$dt = \frac{dv}{a(v)}$$

integrate for $v(t)$; use $v\,dv = a\,dx$ to get x against v

STOPPING DISTANCE

$$s = \frac{u^2}{2|a|}$$

with a reaction time t_r add $u\,t_r$ before braking starts

CHASE AND MEETING

$$x_{rel} = u_{rel}t + \tfrac{1}{2}a_{rel}t^2$$

with $u_{rel} = u_1 - u_2$, $a_{rel} = a_1 - a_2$; they meet when x_{rel} equals the starting gap

HOW IT IS USED

A two-body chase becomes one-body motion with u_{rel} and a_{rel}; no positive root for t means they never meet. **Trap:** $v = u + at$ fails when a changes with time.

5 Relative velocity

Velocity is always measured relative to something. $\vec{v}_{AB}$ is how A moves as seen from B: subtract B's velocity from A's. Rivers, rain, wind and two vehicles are all one vector triangle.

RELATIVE VELOCITY

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

velocity of A as seen from B

RELATIVE SPEED IN A PLANE

$$|\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_Av_B\cos\varphi}$$

φ is the angle between $\vec{v}_A$ and $\vec{v}_B$

CLOSEST APPROACH

$$d_{min} = \frac{|\vec{r}_{AB} \times \vec{v}_{AB}|}{|\vec{v}_{AB}|}$$

$\vec{r}_{AB}$ is the starting separation; they collide only if $d_{min} = 0$

RAIN AND MAN

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

$$\tan\theta = \frac{v_m}{v_r}$$

vertical rain, man walking: $|\vec{v}_{rm}| = \sqrt{v_r^2 + v_m^2}$; θ from the vertical, umbrella tilts towards his motion

AIRCRAFT IN WIND

$$\vec{v}_{ground} = \vec{v}_{air} + \vec{v}_{wind}$$

the same triangle as the river–boat problem

TRAINS AND OVERTAKING

$$v_{rel} = v_1 \pm v_2$$

$$t = \frac{L_1 + L_2}{v_{rel}}$$

minus for the same direction, plus for opposite directions

RIVER–BOAT: SHORTEST TIME

$$t = \frac{d}{v_b}, \text{ drift} = \frac{v_r\,d}{v_b}$$

head straight across; v_b = boat speed in still water, v_r = river speed

RIVER–BOAT: SHORTEST PATH

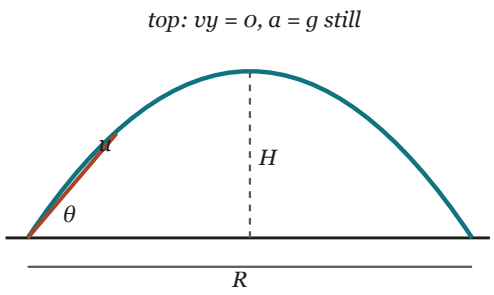
$\sin\theta = \frac{v_r}{v_b}$ (upstream of the normal), $t = \frac{d}{\sqrt{v_b^2 - v_r^2}}$

needs $v_b > v_r$; if $v_b < v_r$ least drift is $\frac{d\sqrt{v_r^2 - v_b^2}}{v_b}$ at $\cos\theta = \frac{v_b}{v_r}$

6 Projectile motion

launch speed u, angle θ with the horizontal

Horizontal and vertical motions are independent: horizontally the speed is constant, vertically the acceleration is g downwards. Time is the only quantity the two share.



$v_x = u\cos\theta$ never changes · $v_y = u\sin\theta - gt$

TIME OF FLIGHT

$$T = \frac{2u\sin\theta}{g}$$

MAXIMUM HEIGHT

$$H = \frac{u^2\sin^2\theta}{2g}$$

RANGE

$$R = \frac{u^2\sin 2\theta}{g}$$

$R_{max} = \dfrac{u^2}{g}$ at 45°

TRAJECTORY

$$y = x\tan\theta\left(1 - \frac{x}{R}\right)$$

same as $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

USEFUL LINKS

$$\tan\theta = \frac{4H}{R}$$

$$R = 4H\cot\theta$$

angles θ and $90^\circ - \theta$ give the same range, with $t_1 t_2 = 2R/g$

HORIZONTAL LAUNCH FROM A HEIGHT

$$t = \sqrt{\frac{2h}{g}},\quad R = u\sqrt{\frac{2h}{g}}$$

landing speed $\sqrt{u^2 + 2gh}$; direction $\tan\beta = \dfrac{gt}{u}$ below horizontal

SPEED AT A GIVEN HEIGHT

$$v^2 = u^2 - 2gy$$

energy conservation, direction not needed

TIME TO REACH A GIVEN HEIGHT

$$h = u\sin\theta\, t - \tfrac{1}{2}gt^2$$

two roots (going up, coming down): their sum is T, their product is $2h/g$

DIRECTION OF VELOCITY

$$\tan\varphi = \frac{v_y}{v_x} = \tan\theta - \frac{gt}{u\cos\theta}$$

φ measured from the horizontal

TRAP

At the top the speed is **not zero** (it is $u\cos\theta$) and a is still g. Minimum speed is at the top.

HOW IT IS ASKED

Often via the trajectory: for $y = ax - bx^2$ read off $\tan\theta = a$, $u^2\cos^2\theta = g/2b$. Or R and H as a ratio.

7 Projectile on an incline

incline angle α, launch angle θ measured from the incline, thrown up the slope

Tilt the axes: x along the slope and y perpendicular to it. The same projectile equations hold, with g split into $g\sin\alpha$ along the slope and $g\cos\alpha$ across it.

TIME OF FLIGHT

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

$a_x = -g\sin\alpha$, $a_y = -g\cos\alpha$

RANGE ALONG THE INCLINE

$$R = \frac{2u^2\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha}$$

down the incline replace α by $-\alpha$

MAXIMUM RANGE

$$R_{max} = \frac{u^2}{g(1+\sin\alpha)}$$ up

$$R_{max} = \frac{u^2}{g(1-\sin\alpha)}$$ down

up the slope at $\theta = 45^\circ - \alpha/2$ from the incline

KEY IDEA

Two projectiles in the air together both accelerate at g, so each sees the other move in a **straight line at constant velocity**. Use relative velocity for mid-air meetings.

8 Uniform circular motion

Velocity changes direction even at constant speed, so a body in circular motion accelerates. That acceleration points to the centre and equals v^2/r; a change in speed adds a tangential part.

ANGULAR QUANTITIES

$$\omega = \frac{d\theta}{dt},\ v = \omega r$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

CENTRIPETAL ACCELERATION

$$a_c = \frac{v^2}{r} = \omega^2 r = v\omega$$

directed towards the centre

NON-UNIFORM CIRCULAR MOTION

$$a_t = \frac{dv}{dt} = \alpha r$$

$$a = \sqrt{a_c^2 + a_t^2}$$

a_t along the tangent, a_c along the radius

ROTATION WITH CONSTANT ACCELERATION

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

number of turns $N = \theta/2\pi$

CHANGE IN VELOCITY

$$|\Delta\vec{v}| = 2v\sin\frac{\theta}{2}$$

over an arc of angle θ; the speed itself does not change

HOW IT IS ASKED

Mostly through angular speed and the centripetal acceleration of a point on a rotating body, or clock hands: convert each hand to its own ω, then to an arc length. In **uniform** motion the average acceleration over one full turn is zero while a_c is never zero.

9 Worked examples

BRAKING WITH REACTION TIME

A car at $20\ \mathrm{m\,s^{-1}}$ brakes at $5\ \mathrm{m\,s^{-2}}$ after a $0.5\ \mathrm{s}$ reaction time. Total stopping distance?

$$20 \times 0.5 + \frac{20^2}{2 \times 5} = 10 + 40 = 50\ \mathrm{m}$$

DISTANCE AND DISPLACEMENT

A particle has $x = t^3 - 6t^2 + 9t$ (SI). Distance and displacement in the first $3\ \mathrm{s}$?

$v = 3(t-1)(t-3)$, zero at $t = 1, 3$; $x(1) = 4$, $x(3) = 0$: distance $8\ \mathrm{m}$, displacement 0

PROJECTILE

A ball is thrown at $20\ \mathrm{m\,s^{-1}}$ at 30° above the horizontal, $g = 10\ \mathrm{m\,s^{-2}}$. Find T, H, R.

$$T = 2\ \mathrm{s},\ H = 5\ \mathrm{m},\ R = 20\sqrt{3} \approx 34.6\ \mathrm{m}$$

RIVER CROSSING

River $4\ \mathrm{m\,s^{-1}}$, boat $5\ \mathrm{m\,s^{-1}}$ in still water, width $100\ \mathrm{m}$. Shortest time and shortest path?

Time $20\ \mathrm{s}$ (drift $80\ \mathrm{m}$). Path: $\sin\theta = \frac{4}{5}$, $t = \dfrac{100}{3} \approx 33.3\ \mathrm{s}$

10 Last-minute checklist

- Fix a sign convention (up or right is positive) before you substitute into $v = u + at$.
- Distance in the nth second is s_n; distance in n seconds is s. Read the question twice.
- For graphs, ask for slope first, then area. Units give the answer away.
- Projectile: split into x and y; time is the only thing they share.
- River problems: pick the quantity being minimised (time or drift) before choosing the heading.

Now test it. Practise this chapter with past-paper and bank questions on Prep Vault. **Practice this chapter →**